Design and Implementation of Speech Recognition Systems

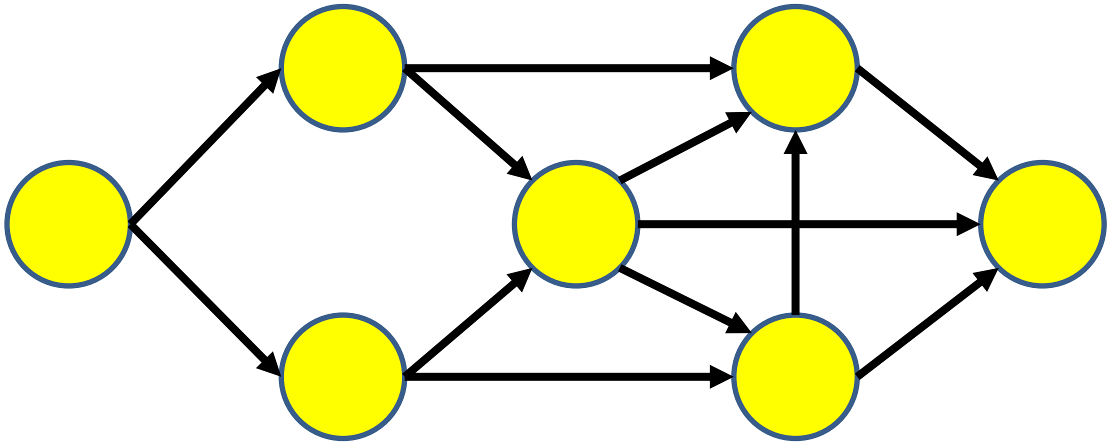
Spring 2013

Class 27: Rescoring, Nbest and Confidence
29 Apr 2013

Topics

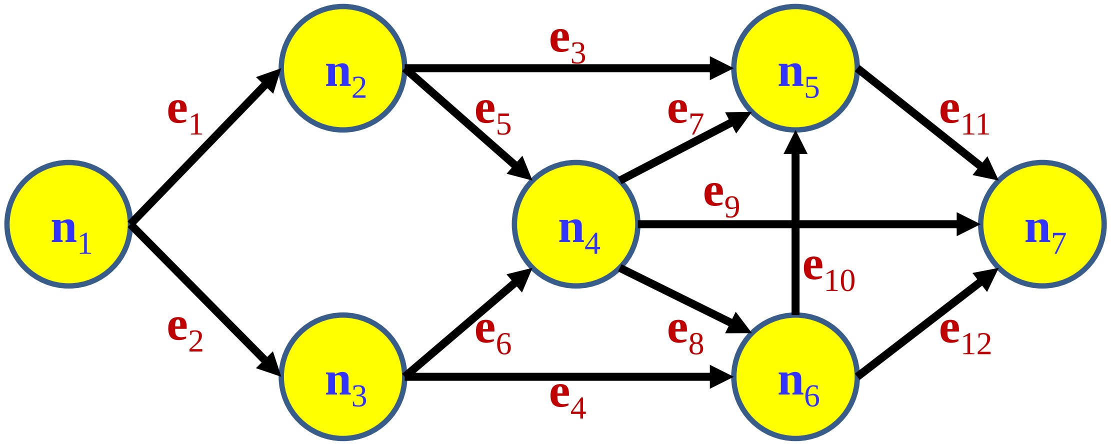
- The backpointer table as a directed acyclic graph
- N-best path search through a graph
  - Stack decoder
  - A*
- Confidence estimation
  - Forward Backward algorithm
- Acoustic Rescoring

The *Directed Acyclic Graph*



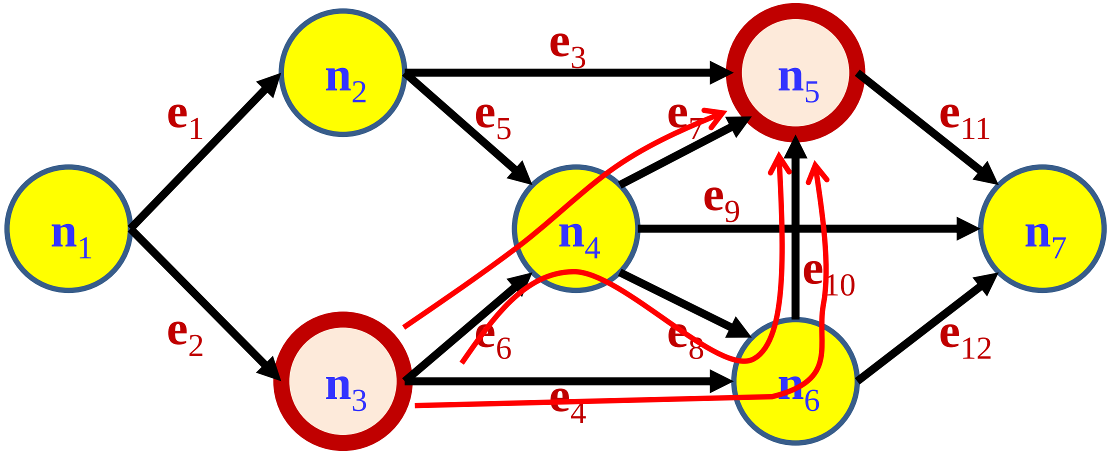
- A graph where every edge has a direction
- There are *no loops*
  - There is no path that revisits a node
- Such a graph *must* have some nodes that are purely ***source*** nodes
  - No incoming edges
- It also *must* have some nodes that are purely ***sink*** nodes
  - No outgoing edges

The *Directed Acyclic Graph*



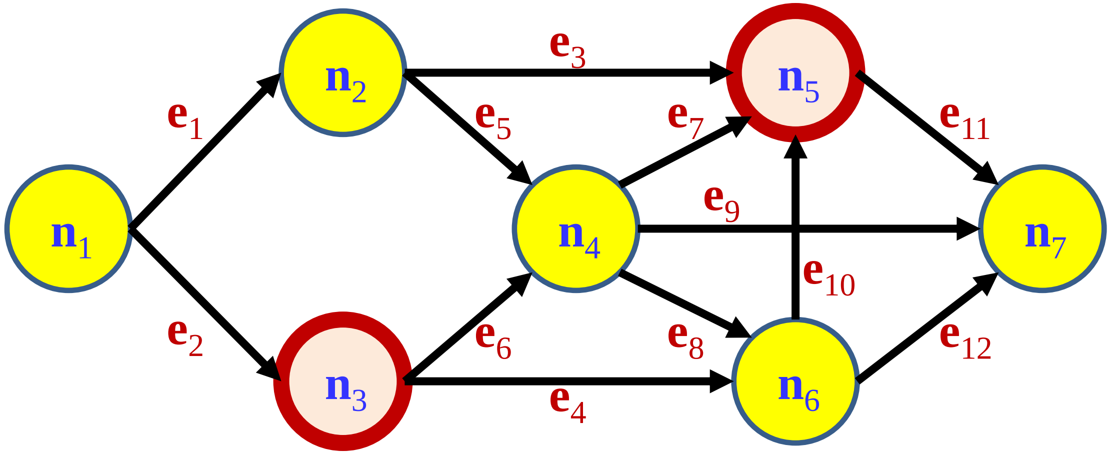
- A graph where every edge has a direction
- Nodes may node ***node cost***
- Edges may have ***edge cost***
- Strictly equivalent: a graph with only edge costs
  - Node costs “pushed” onto edges

The *shortest-path* problem



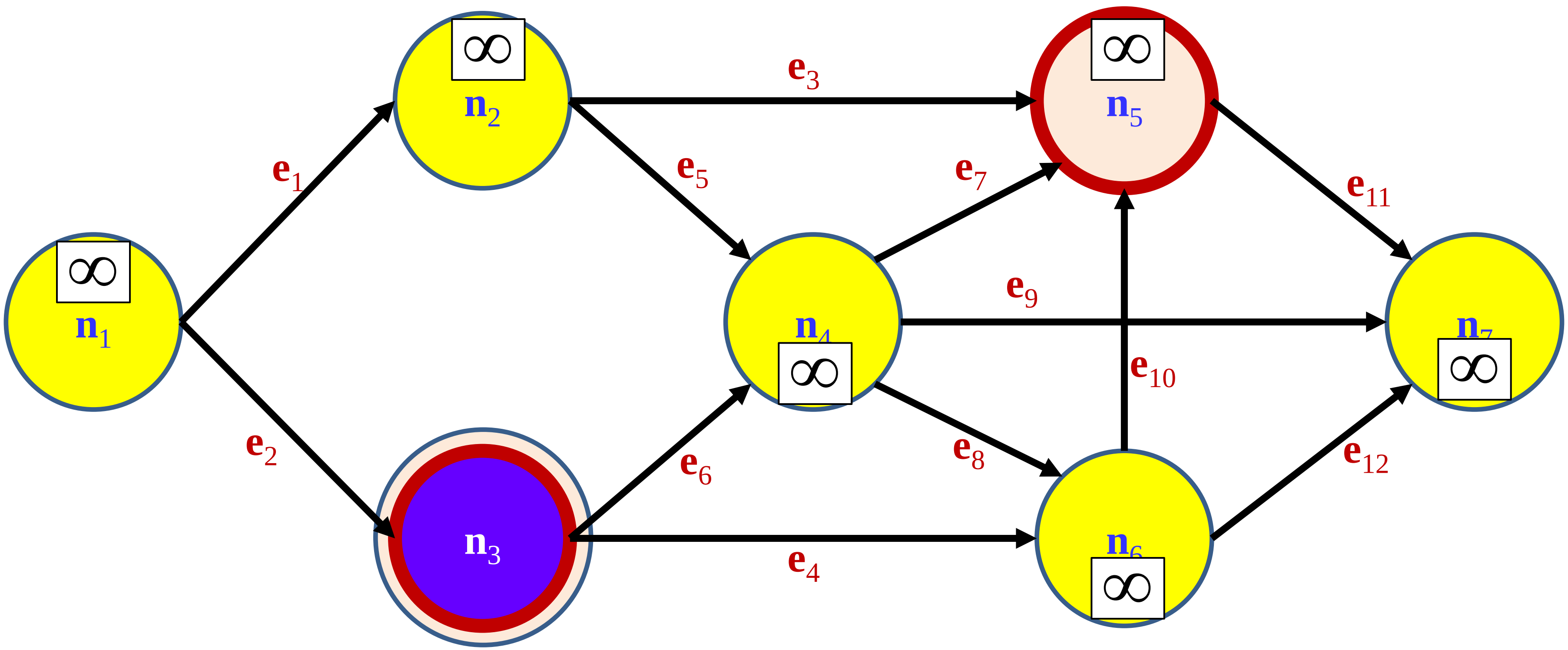
- What is the shortest path from one node to another?
  - "Shortest" ➔ least-cost
  - Could also mean *most-score*
    - If values assigned to nodes and edges represent scores instead of costs
    - We'll assume costs for now; easily modified to deal with scores
      - Simply flip "min" to "max" and vice versa

Dijkstra's algorithm (1959)



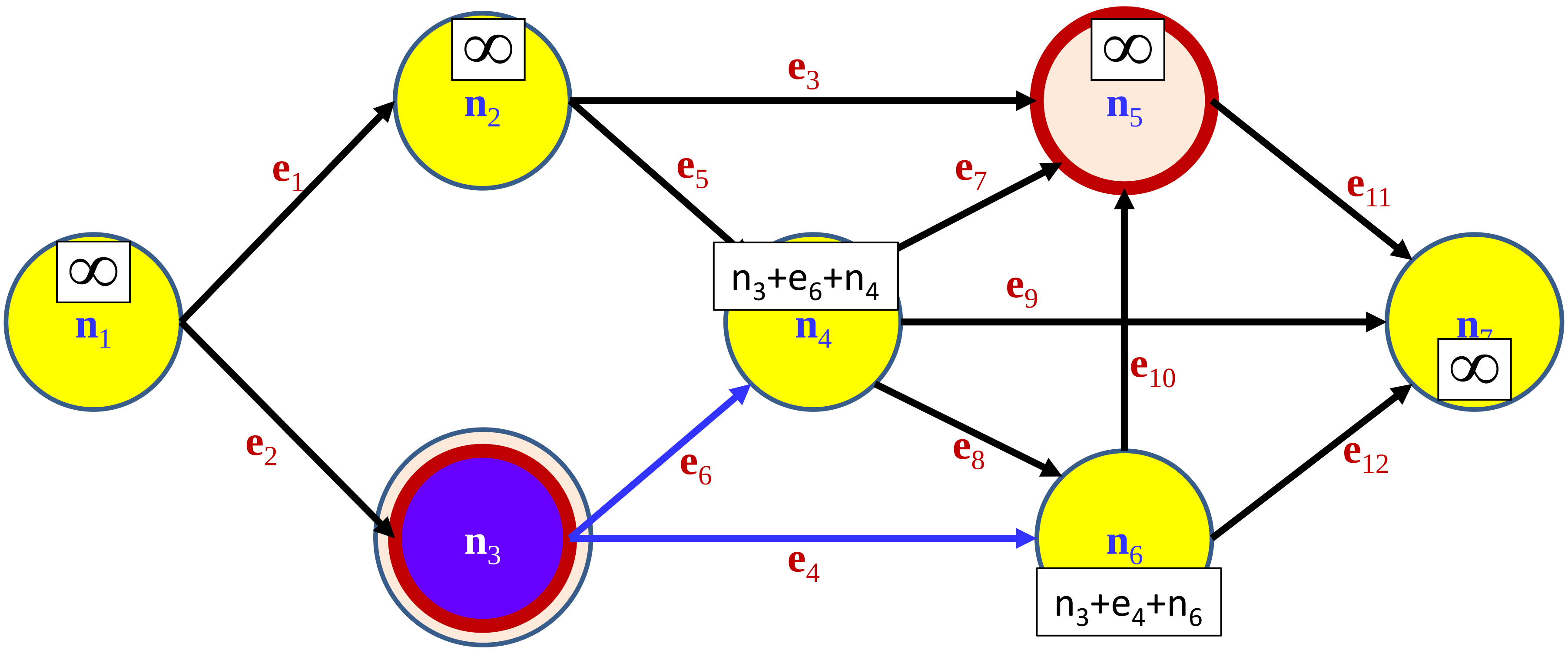
- Gives the cost of the shortest path between two nodes
- Condition: All costs are positive
  - I.e. traversing an edge is expensive
  - Addendum: visiting a node is expensive
    - For a DAG, node costs can be converted to edge costs by simply adding them to all outgoing edges from that node
  - Passing through a node or edge can never be profitable
    - Can never have negative cost

Dijkstra's algorithm



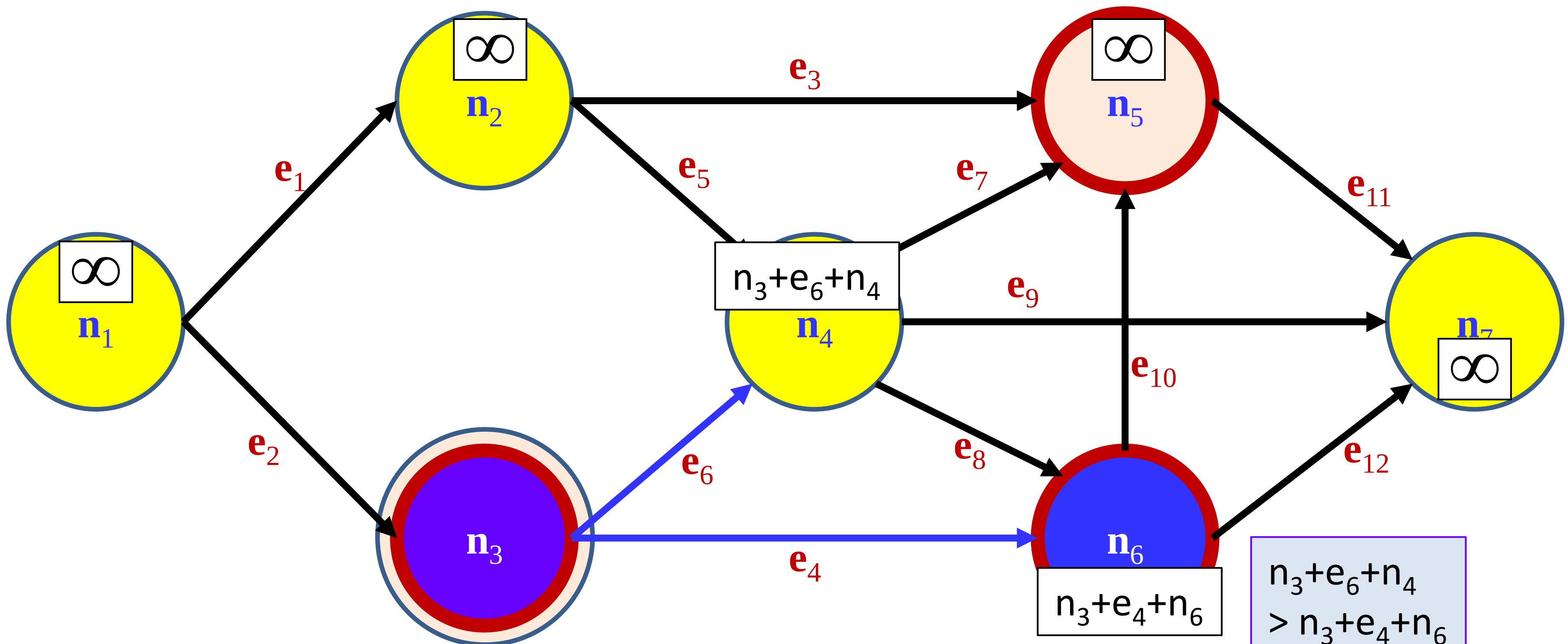
1. **Set current node to "visited" state**
   - Indicated by blue color
   - Cost of best path to current node is simply node cost
   - Set best path cost of *all other nodes* to infinity

Dijkstra's algorithm



1. Set current node to “visited” state
2. **Extend paths from current node to all of its unvisited children**
   - Add edge cost + node cost to get “current” path cost
   - If current path cost to a node is ***lower*** than existing path cost, replace existing path cost with current path cost

Dijkstra's algorithm



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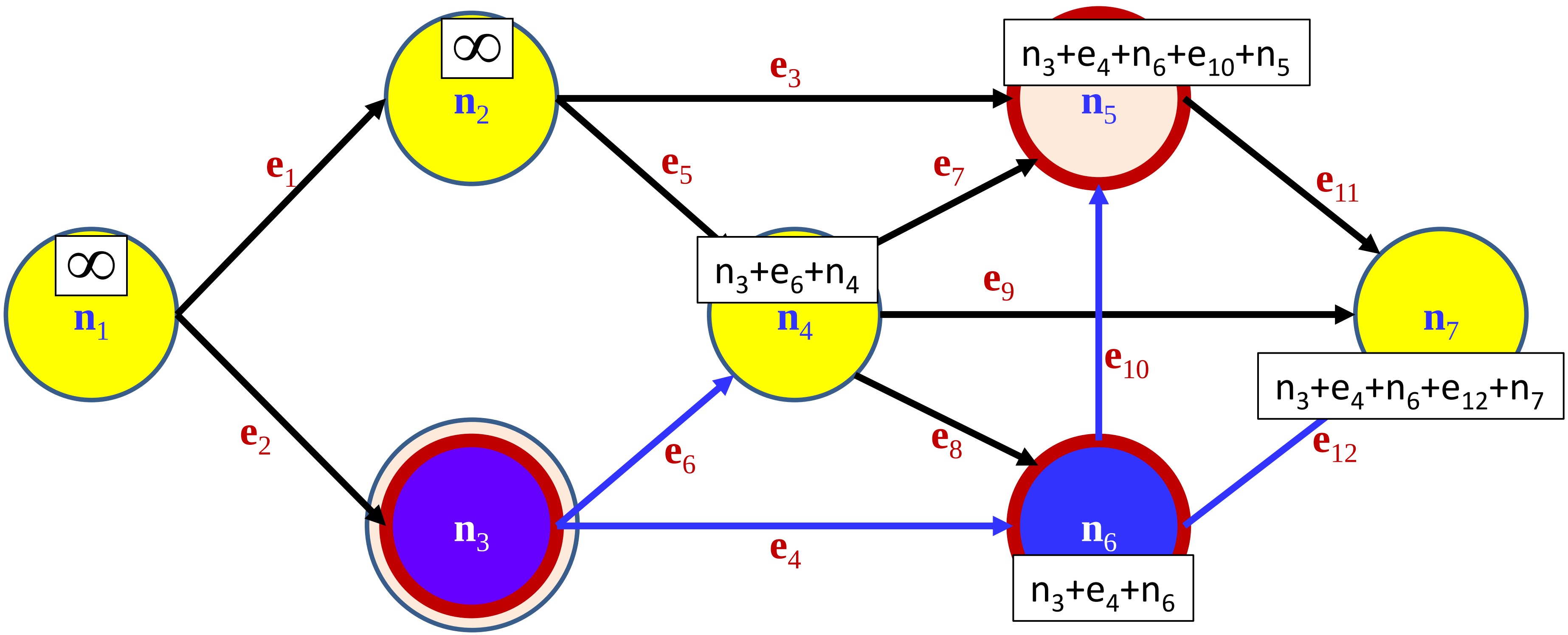
3. **Select the “unvisited” node with lowest cost: set it to “visited”**
   - **If this is the destination node, terminate; shortest path cost found**
   - If the lowest cost unvisited node has a cost of infinity, fail (no path).

Dijkstra's algorithm



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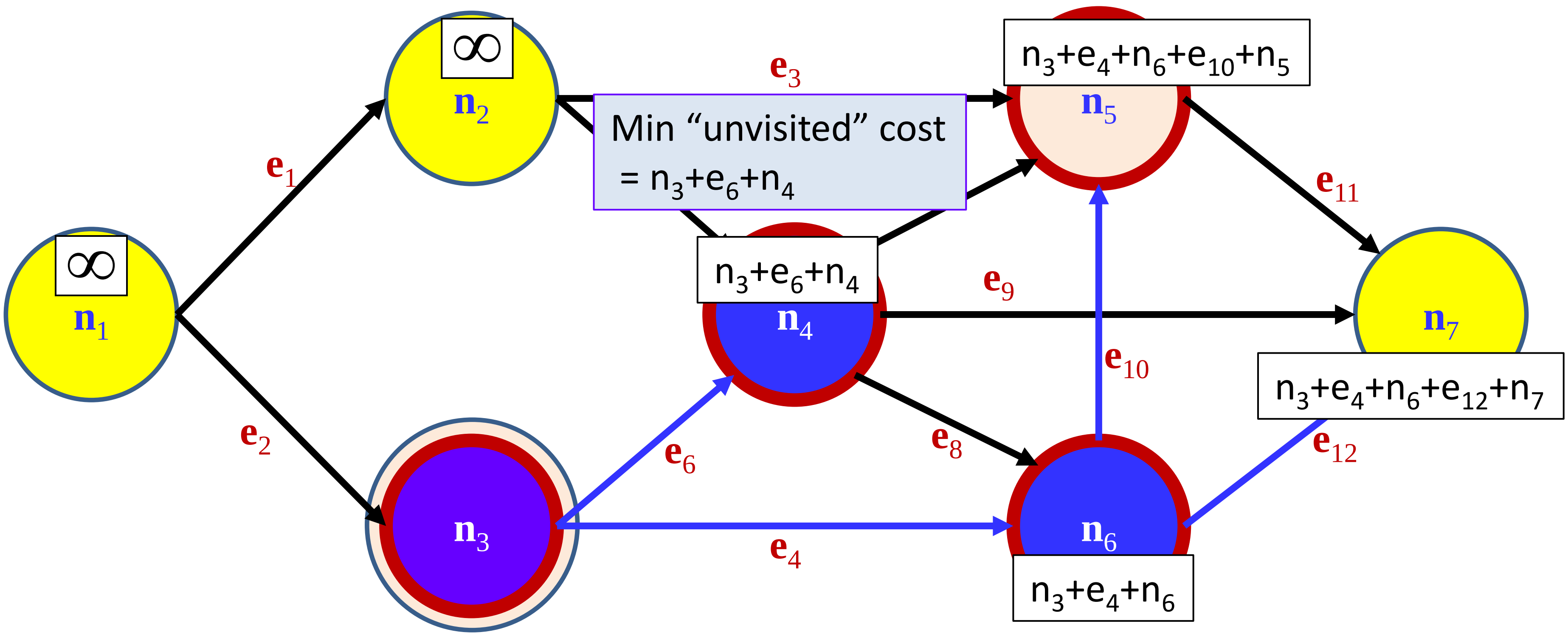
4. Set the node to "current" and return to 2

Dijkstra's algorithm



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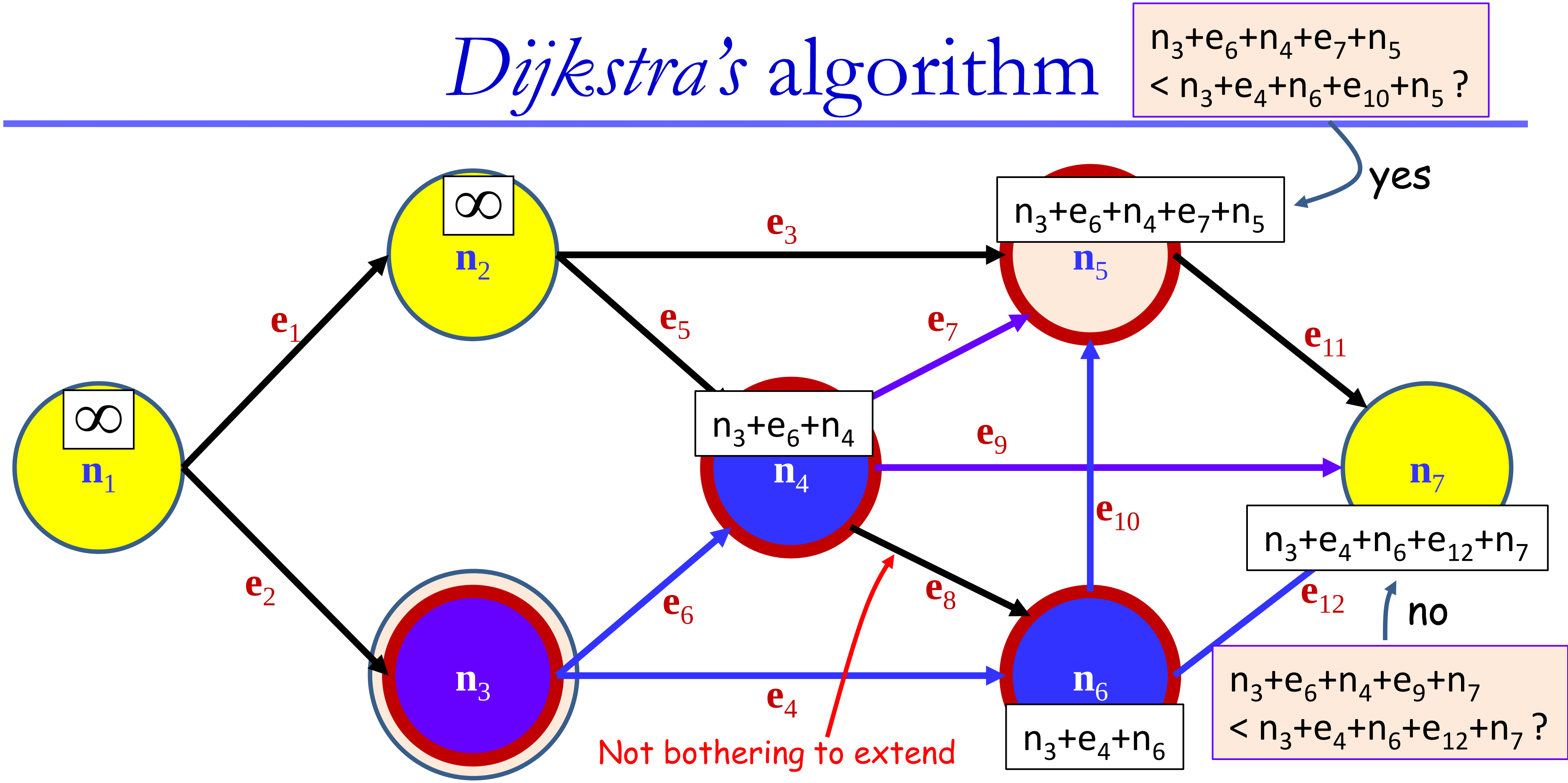

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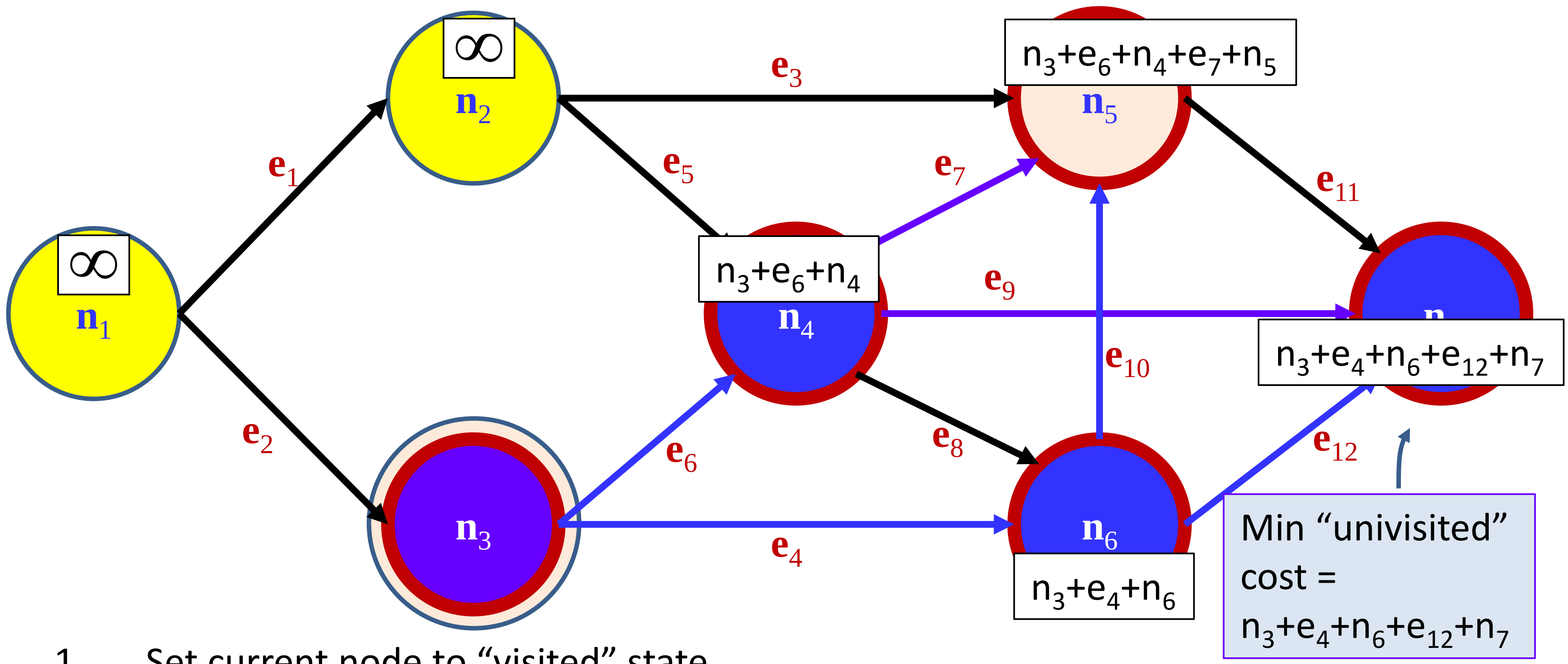

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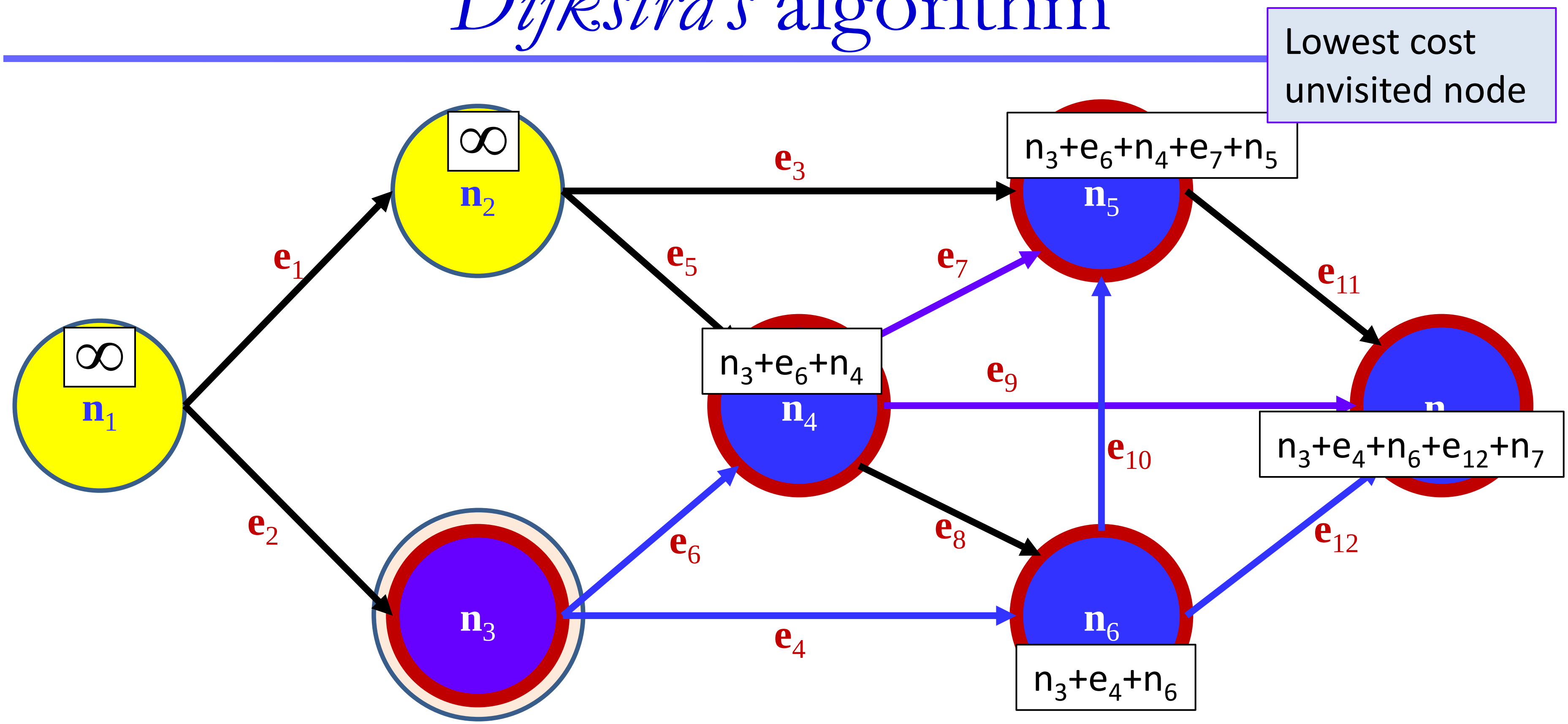

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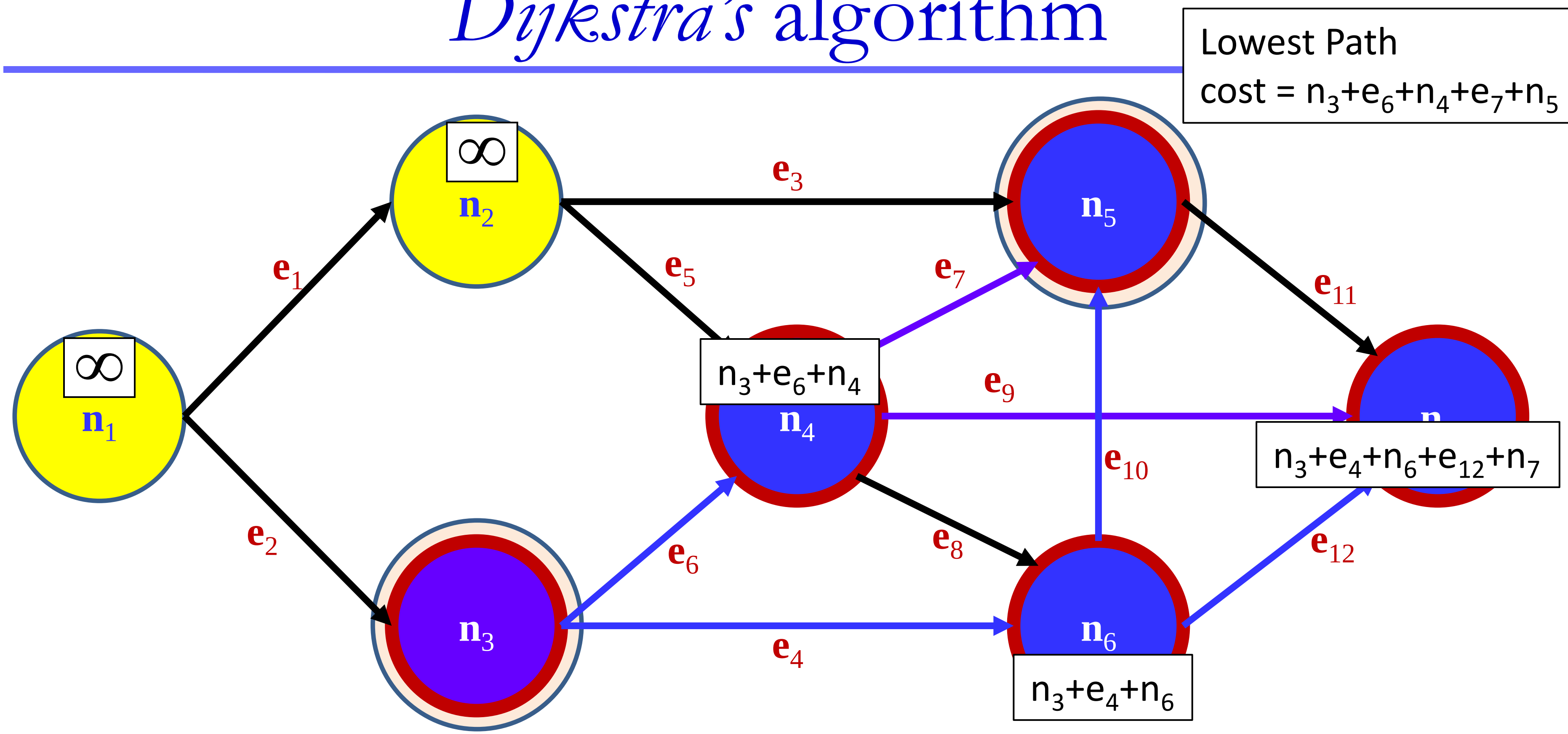
Dijkstra's algorithm



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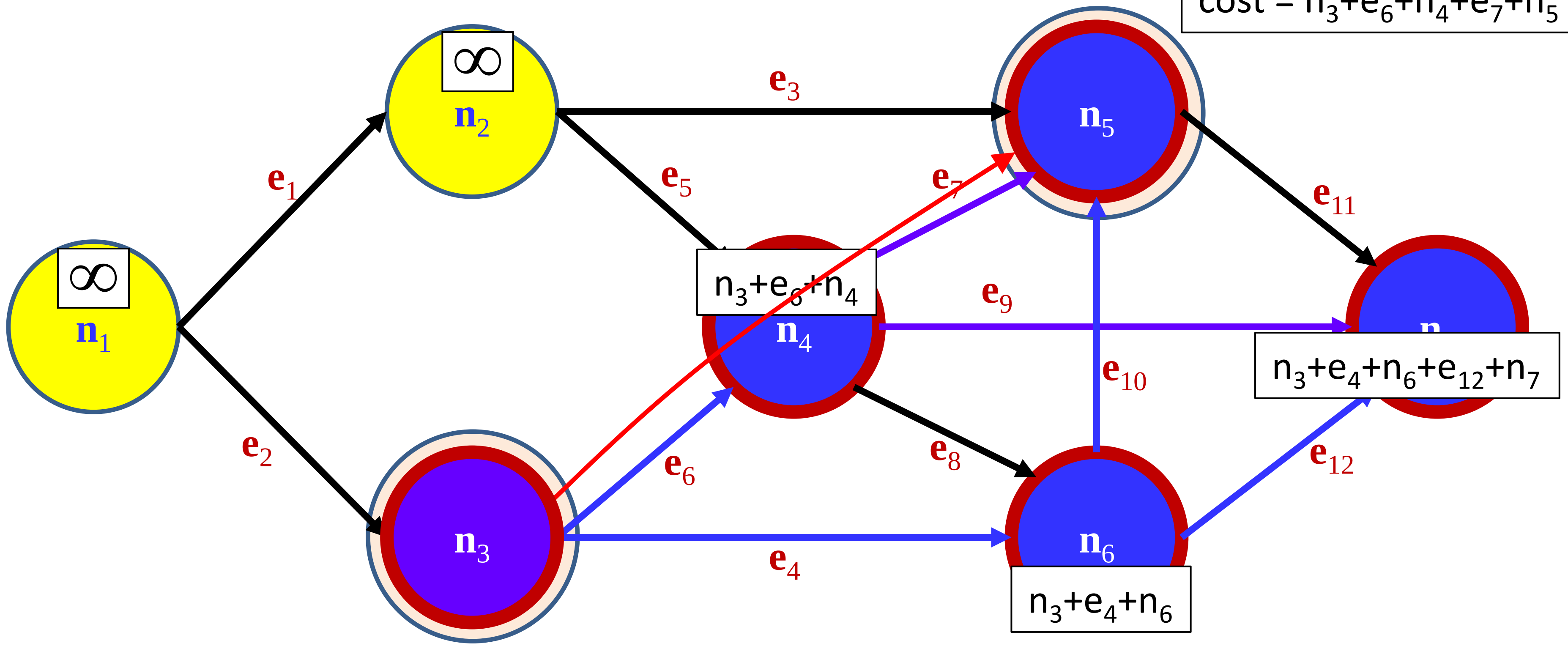
Dijkstra's algorithm

Lowest Path
cost = $n_3+e_6+n_4+e_7+n_5$



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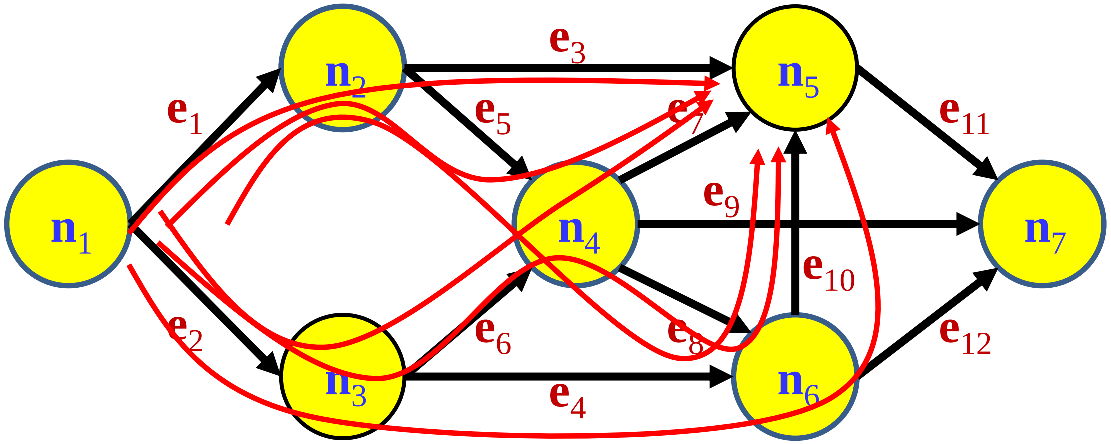
Dijkstra's Algorithm

- Dijkstra's algorithm can be continued to find the shortest path score from the source node to *all* nodes in the graph

- Simply continue algorithm until either
  - All nodes are visited OR
  - All unvisited nodes have infinite cost

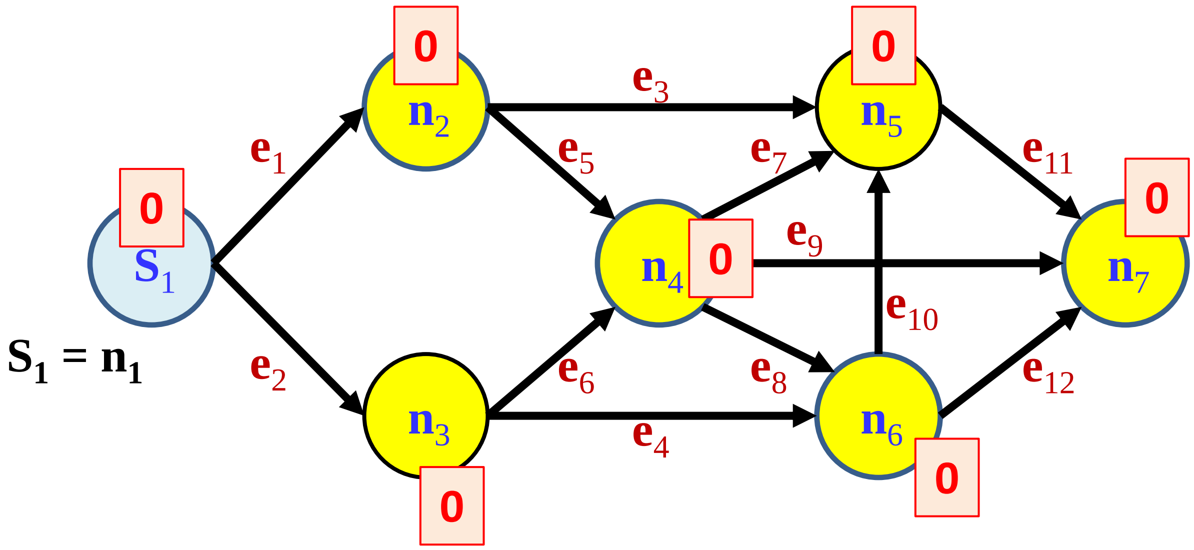
- Computational Cost
  - Naïve implementation $|V|^2$
    - $|V|$ = no. of nodes
  - Optimal implemantation: $|E| + |V| \log |V|$
    - $|E|$ = no. of edges

Problem 2: Computing *Total* path cost



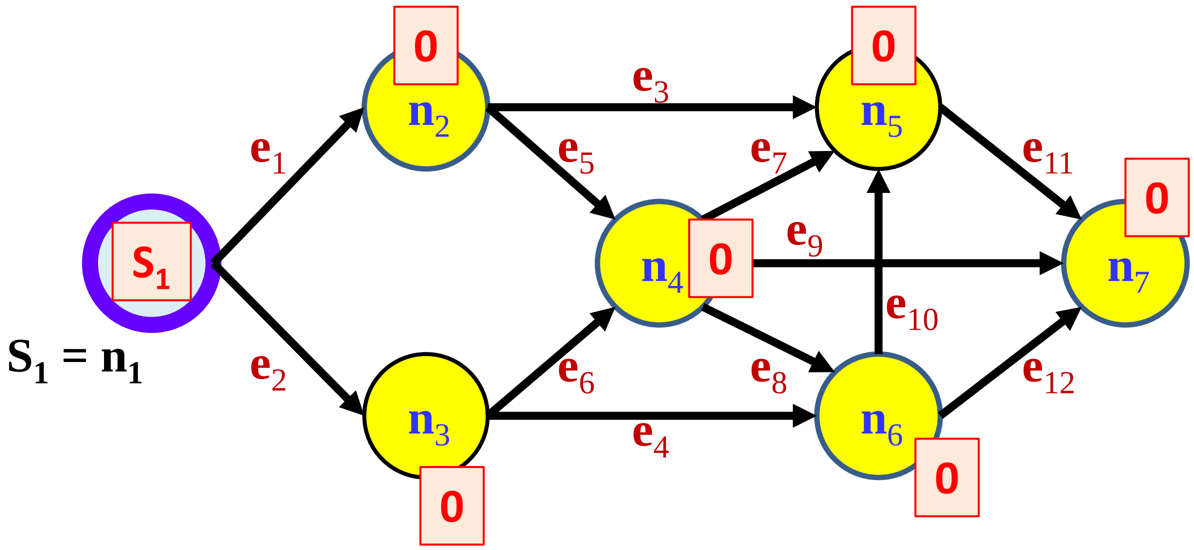
- What is the *total* cost of all paths from all source nodes to any particular node?

Problem 2: The forward algorithm



- Initialize: Set "total path score" for all nodes to 0

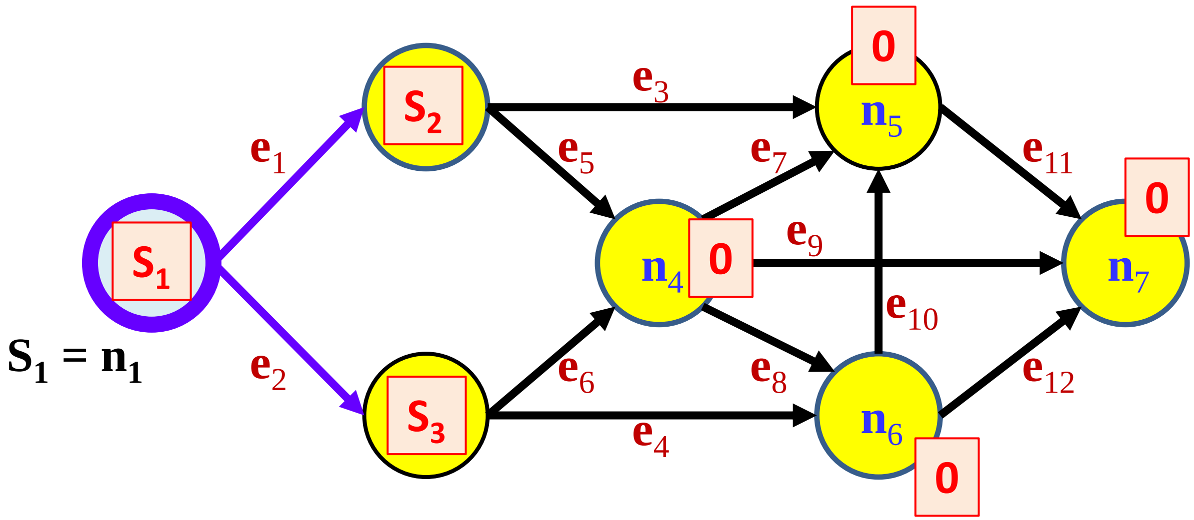
Problem 2: The forward algorithm



1. Mark source nodes

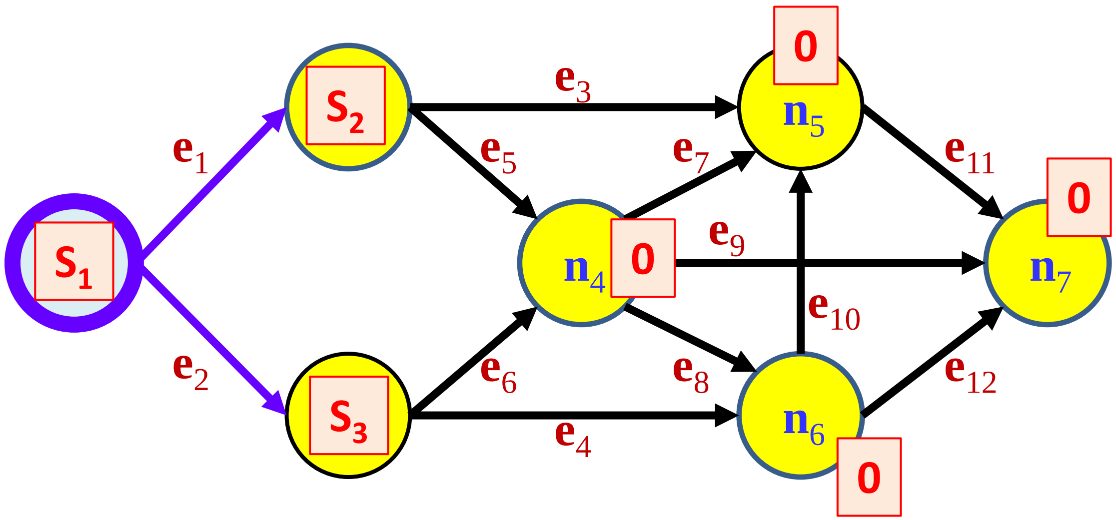
– Source nodes have node scores

Problem 2: The forward algorithm



1. Mark source nodes
2. **Extend paths from all source nodes to all children nodes**
   – Update node scores

Updating Node Scores



- Extending a path: Cost of extended path:
  - Path score = f_{ext}(current path score, edge score, node score)
  - Typically $f_{ext}(a,b,c) = a+b+c$ or $a*b*c$
  - **If edge and node scores are probabilities, we use a*b*c**
- Converging paths: If K paths converge on a node, node score is:
  - node score = node score + f_{node}(path score1, path score2)
  - **If node and edge scores are probabilities, we use $f_{node}(a,b,c) = a+b+c$**

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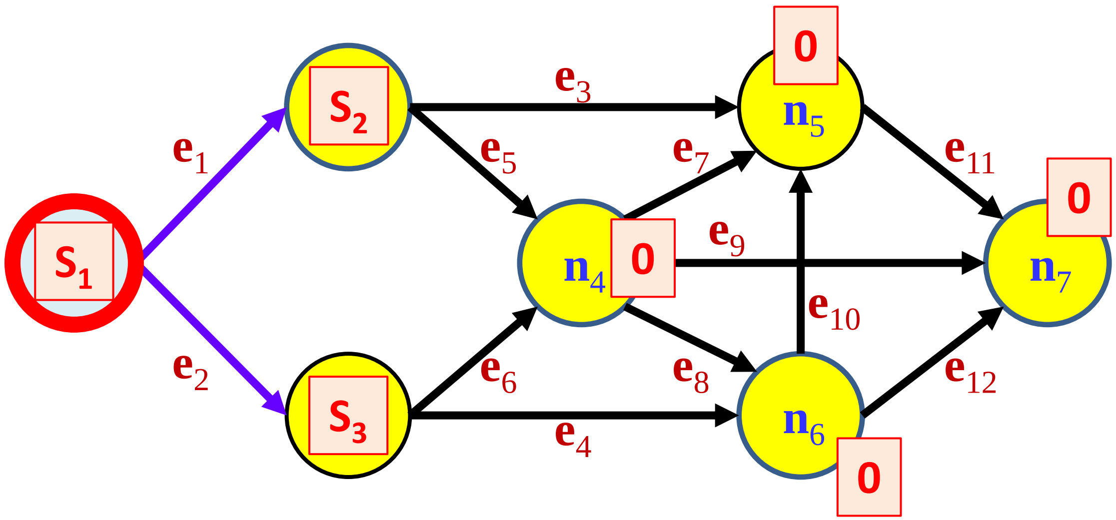

1. Mark source nodes
2. Extend paths from all source nodes to all children nodes
3. **Mark utilized sources and edges as "evaluated"**
   - Mark all utilized edges as "evaluated"
   - Mark all current source nodes as "evaluated"

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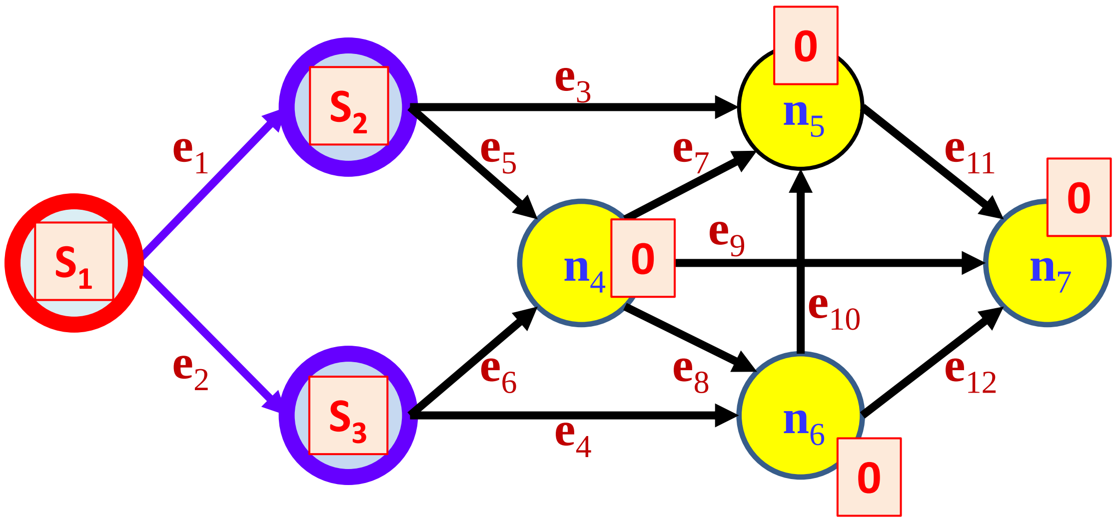

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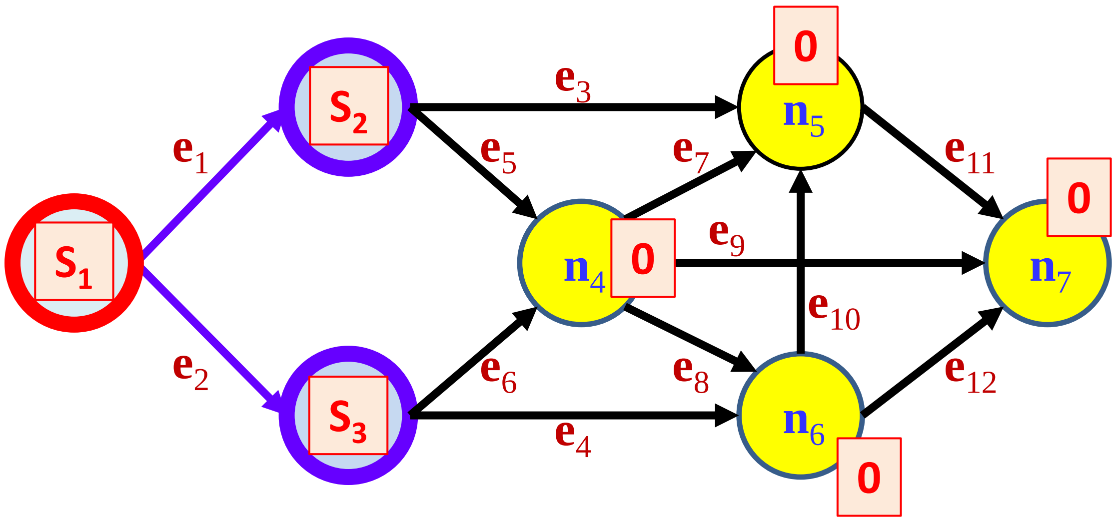

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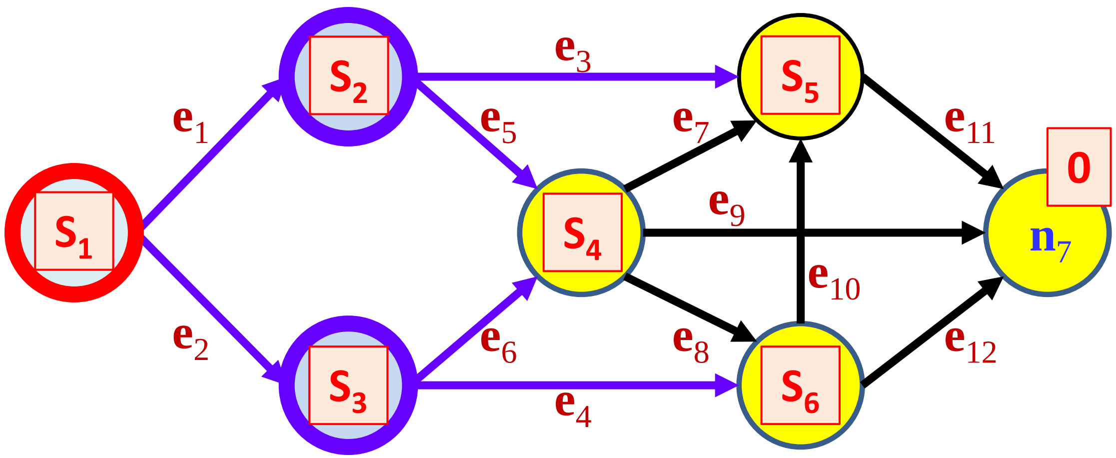

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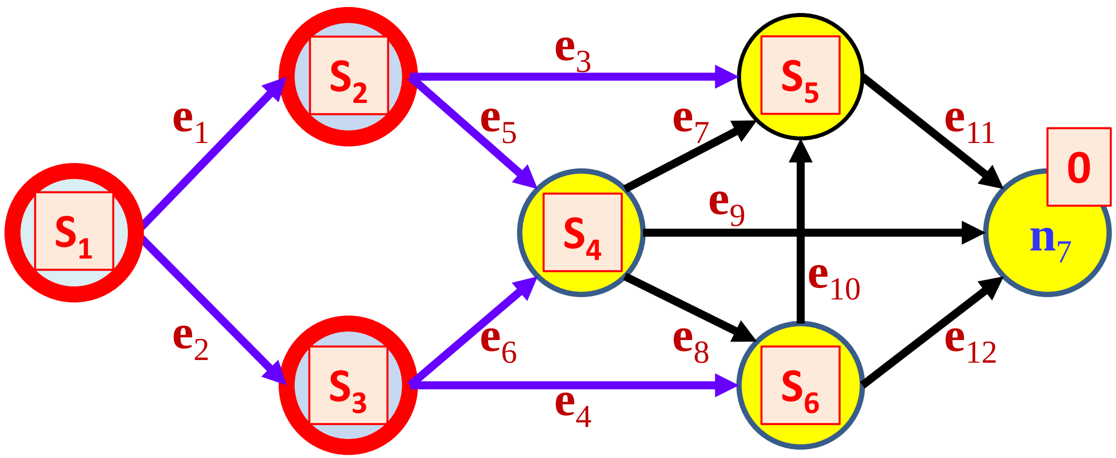

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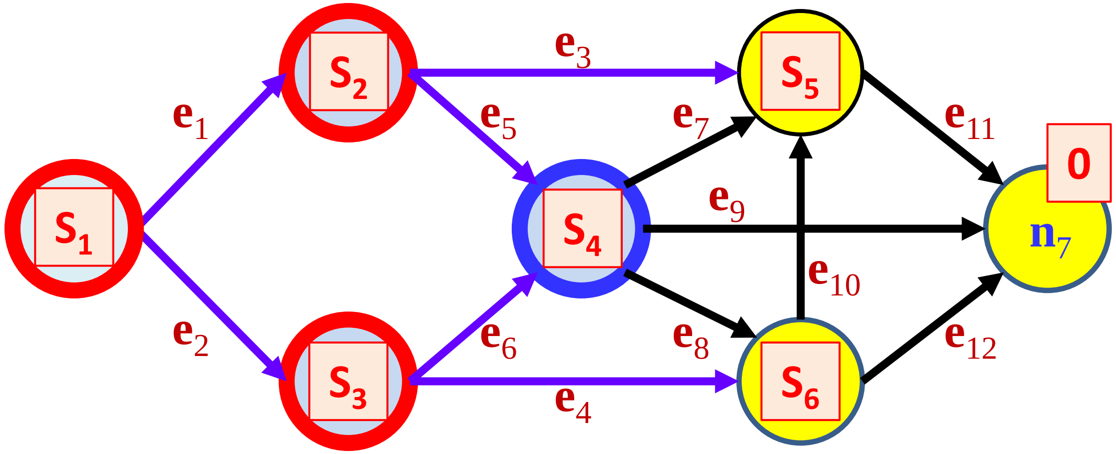

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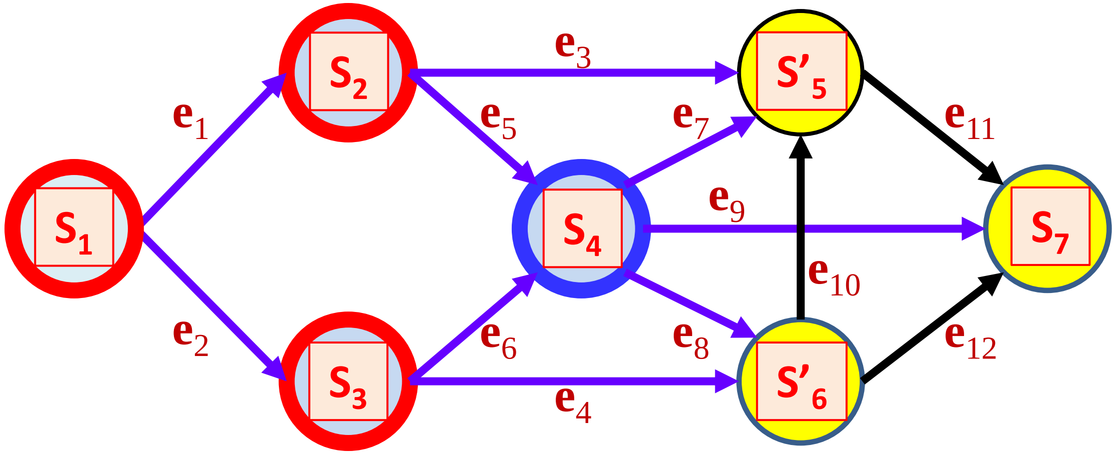

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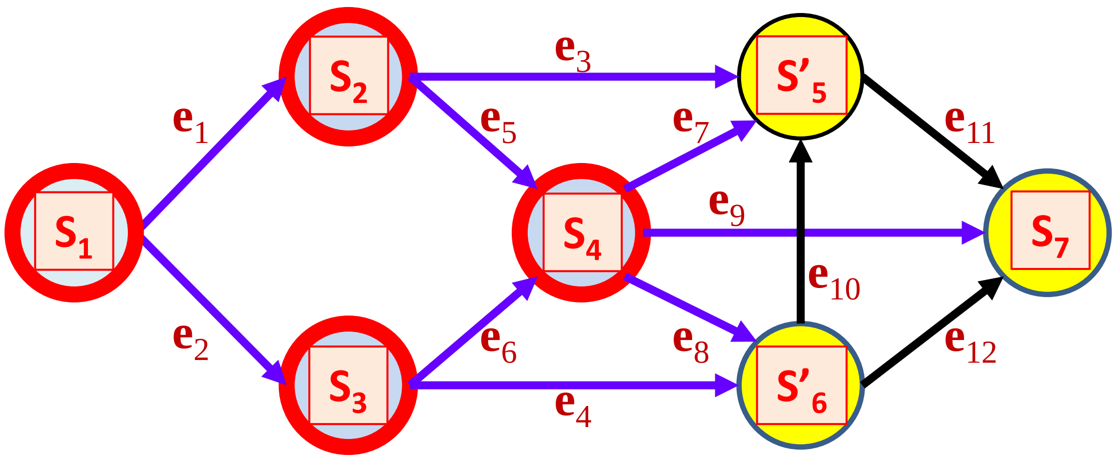

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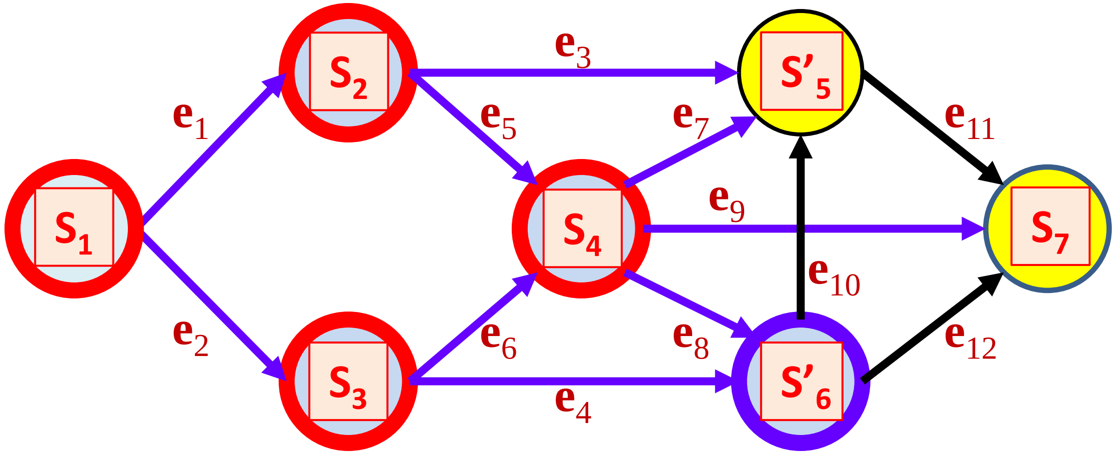

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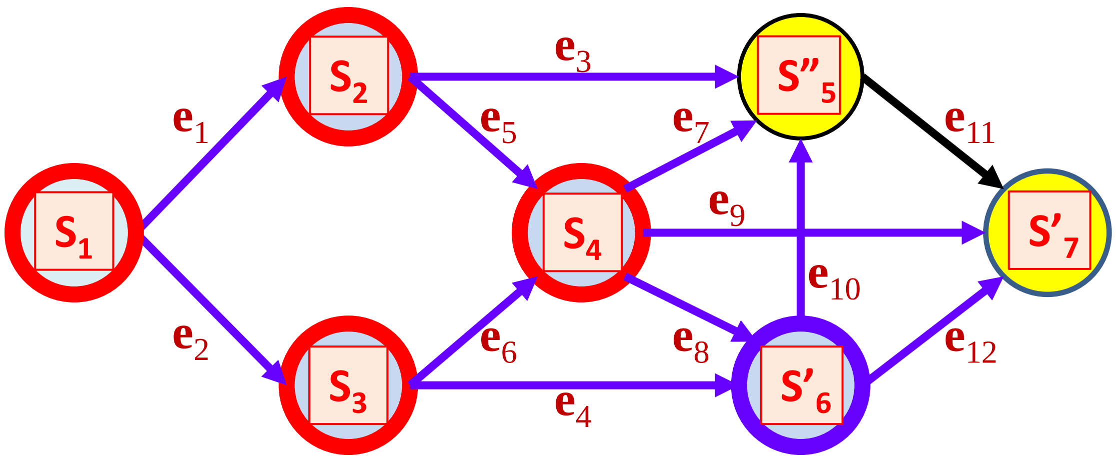

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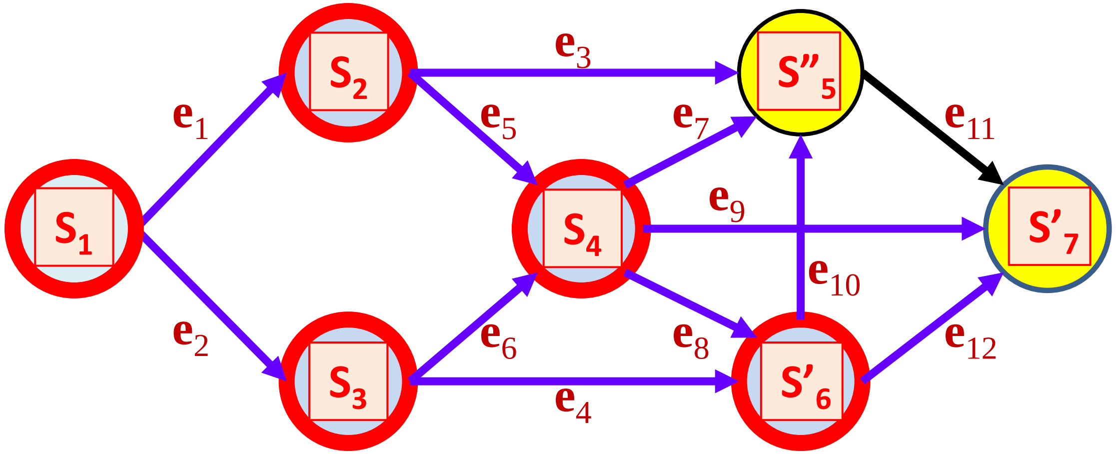

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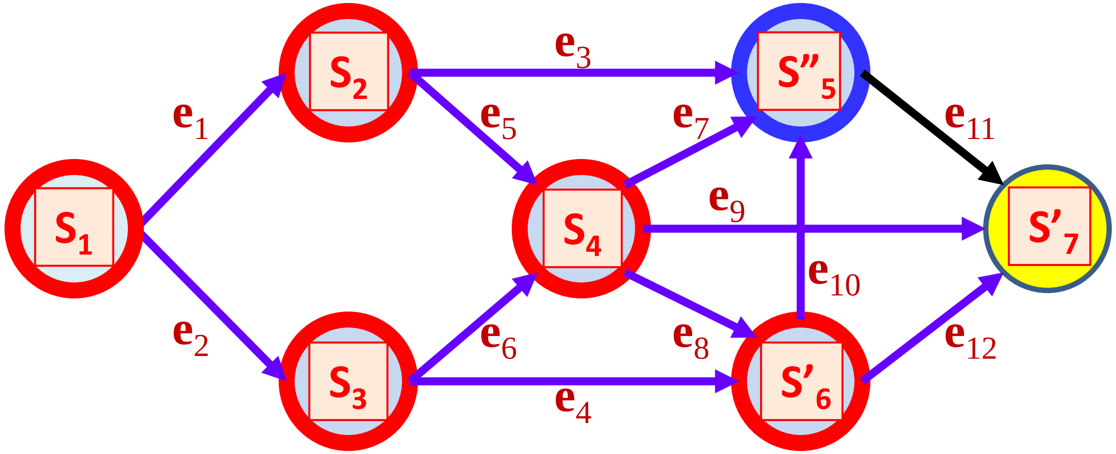

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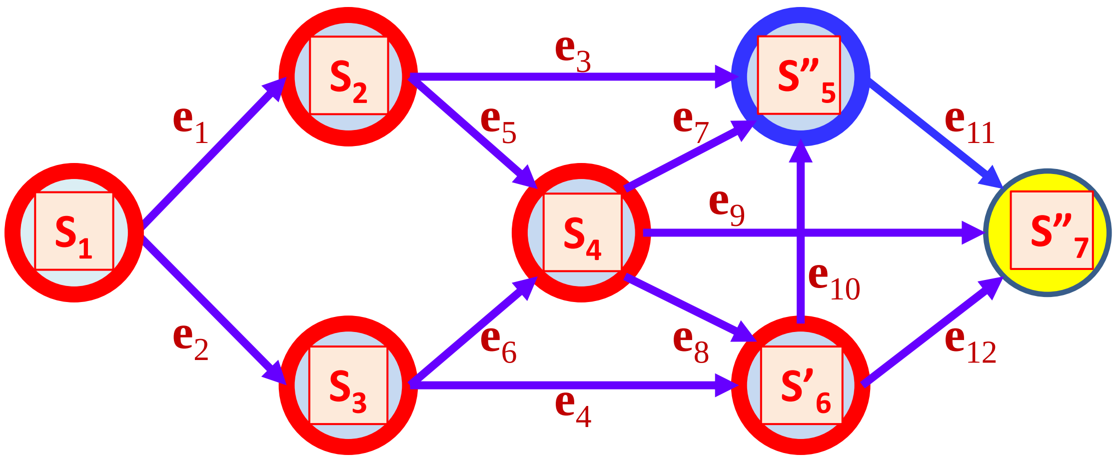

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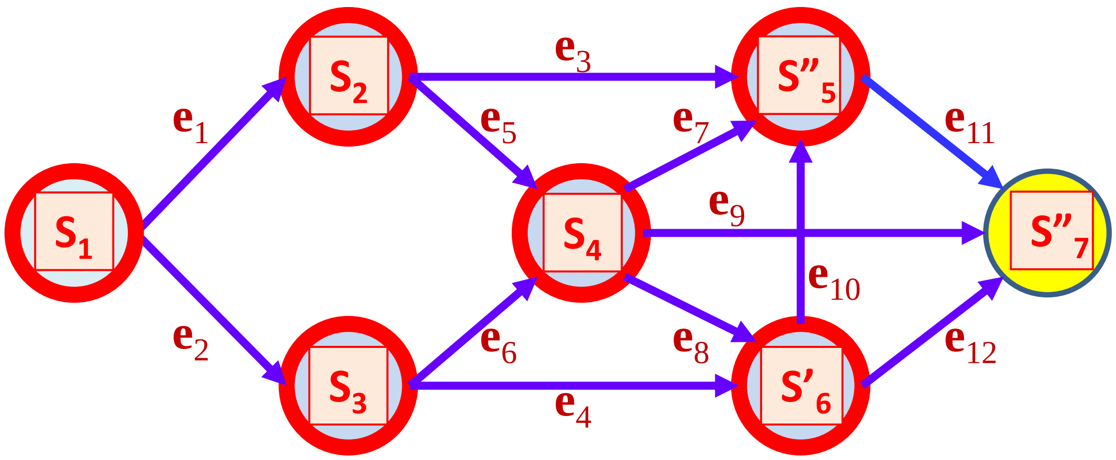

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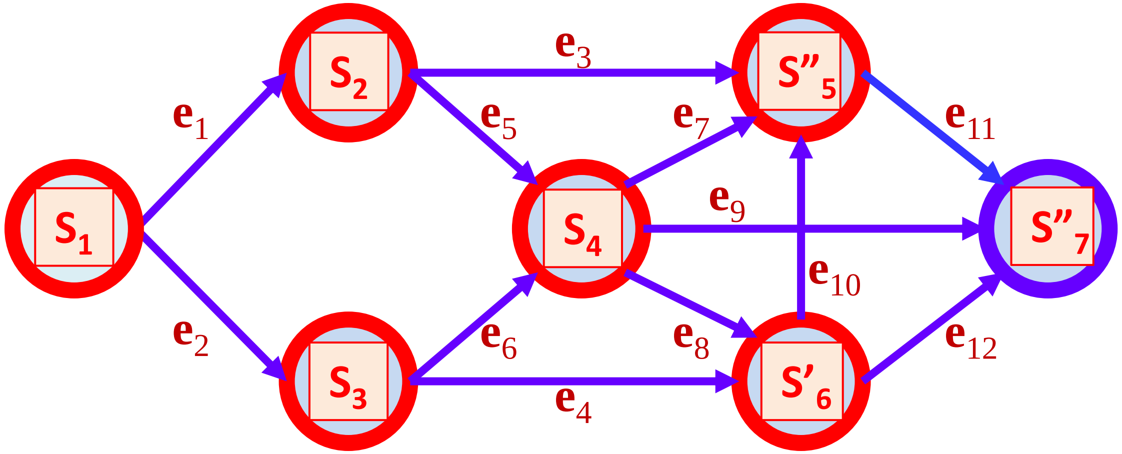

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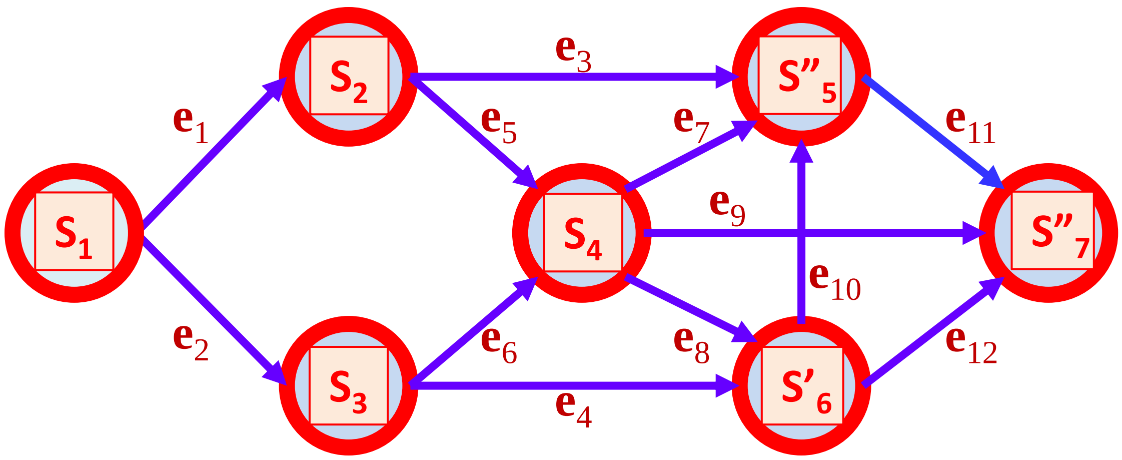

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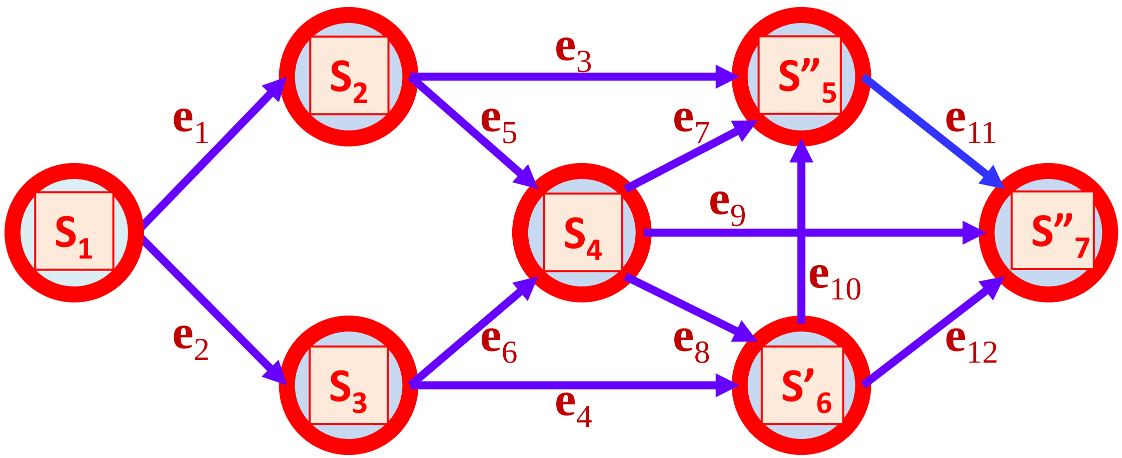
The Forward Algorithm

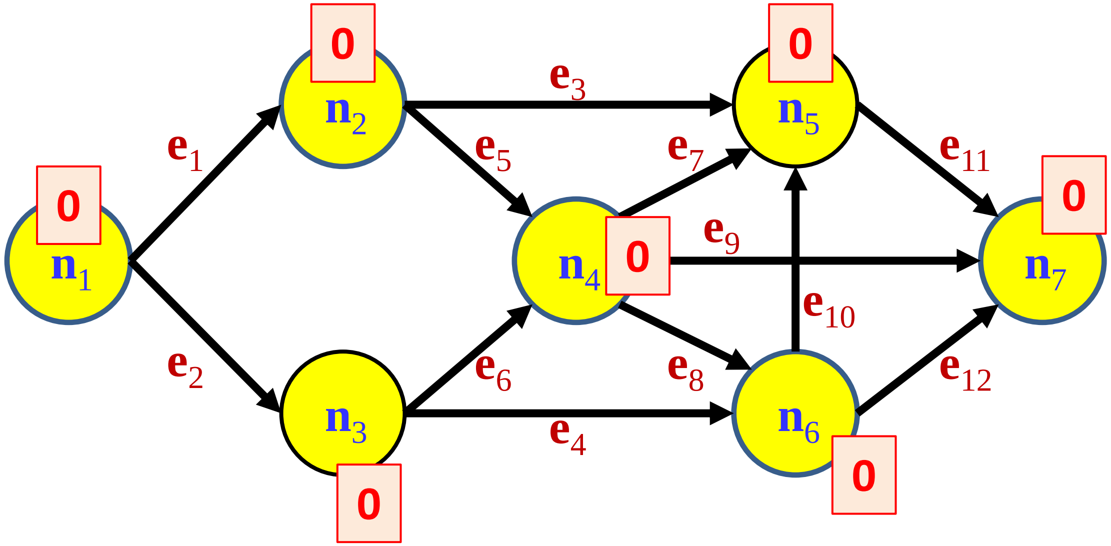


- **At termination**: The final score of any node is the total path score of *all* paths from all source nodes to that node
- The total score of *all* sink nodes is the total score of *all* paths through the graph

The Backward Scores

- The forward algorithm computes total score of all paths from sources to any node
- We can similarly compute the total score of all paths *from a node* to all sink nodes
- This is computed using a *backward* algorithm

Problem 3: The backward algorithm



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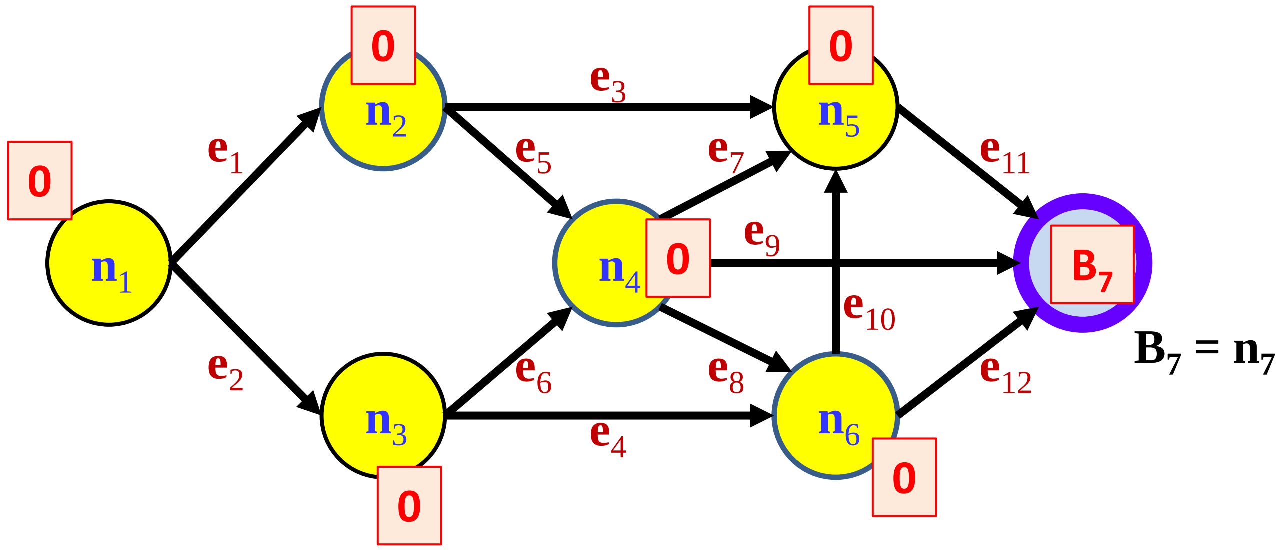
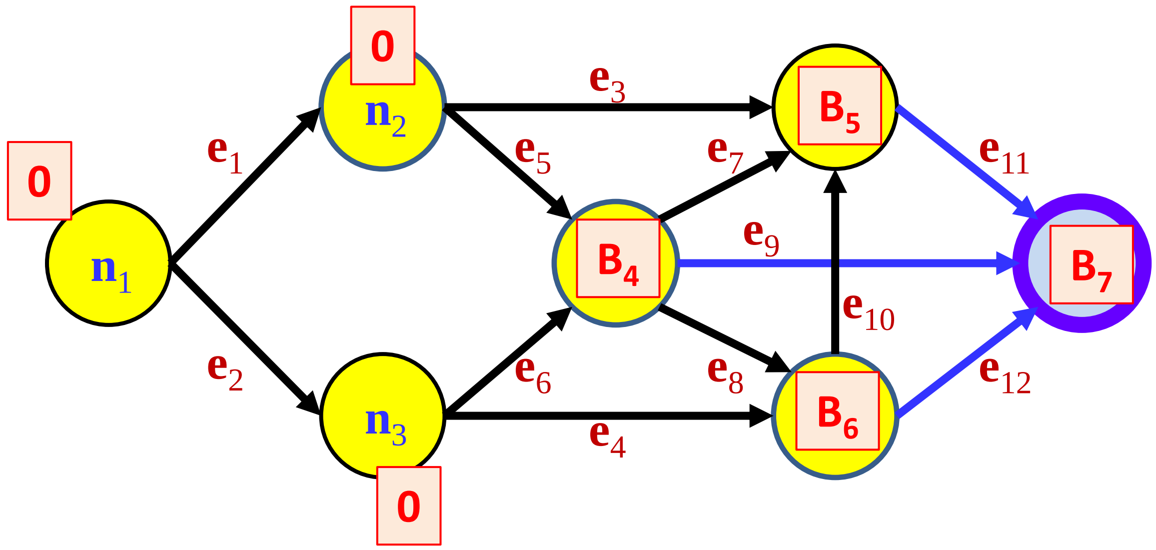
Problem 3: The backward algorithm



1. Mark *sink* nodes

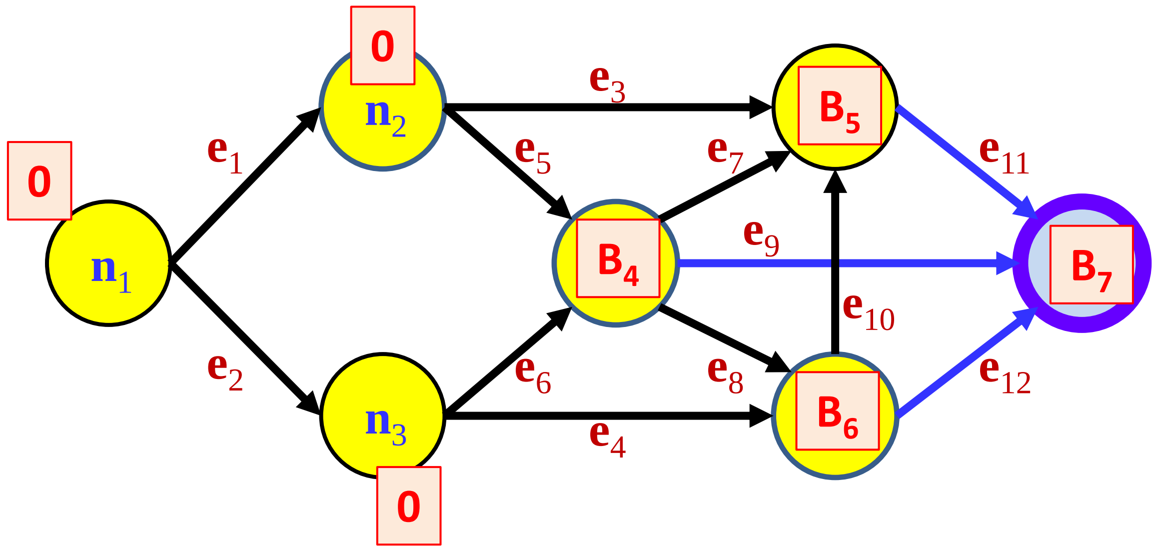
– Sink nodes have node scores

Problem 3: The backward algorithm



1. Mark sink nodes
2. **Extend paths *backwards* from all sink nodes to all *parent* nodes**
   - Update node scores similarly to the forward algorithm

Problem 3: The backward algorithm



- Extending a path: Cost of extended path:
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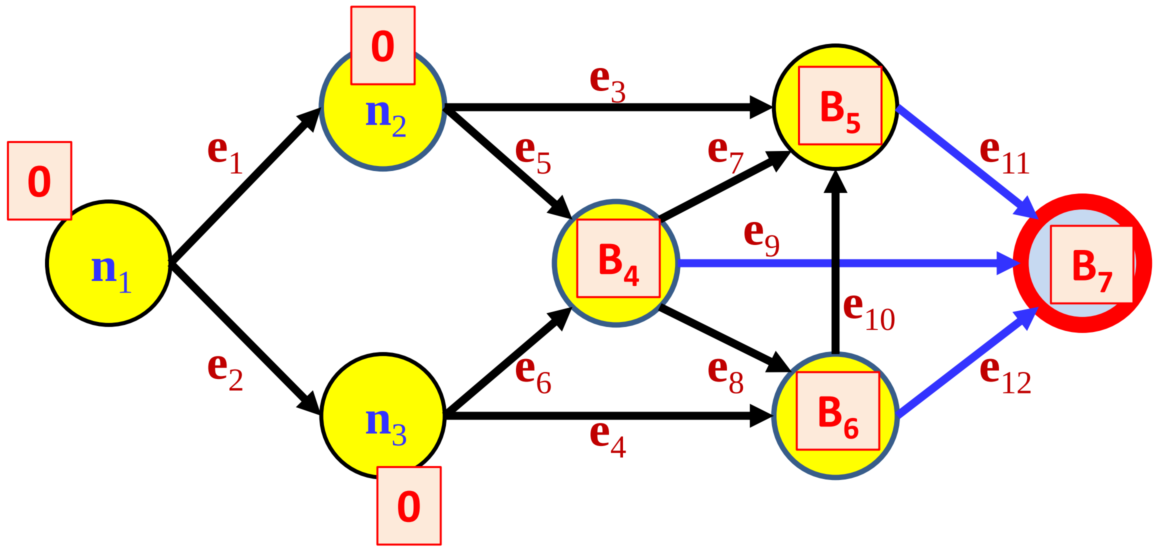
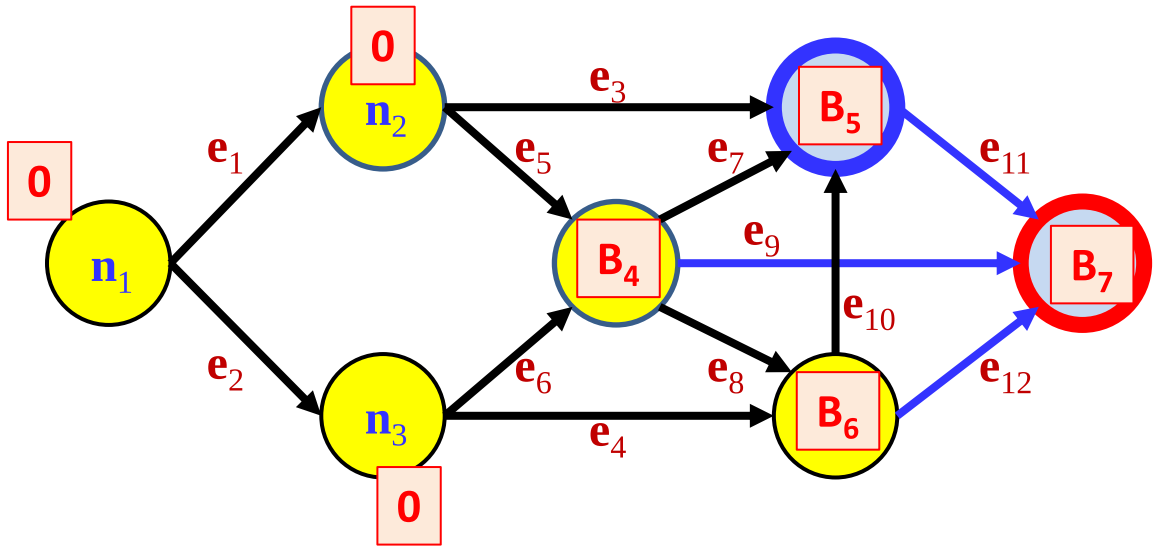
  - Typically $f_{ext}(a,b,c) = a+b+c$ or $a*b*c$
  - **If edge and node scores are probabilities, we use a*b*c**
- Converging paths: If K paths converge on a node, node score is:
  - node score = node score + f_{node}(path score1, path score2)
  - **For probabilistic graphs, $f_{node}(a,b,c) = a+b+c$**

Problem 3: The backward algorithm



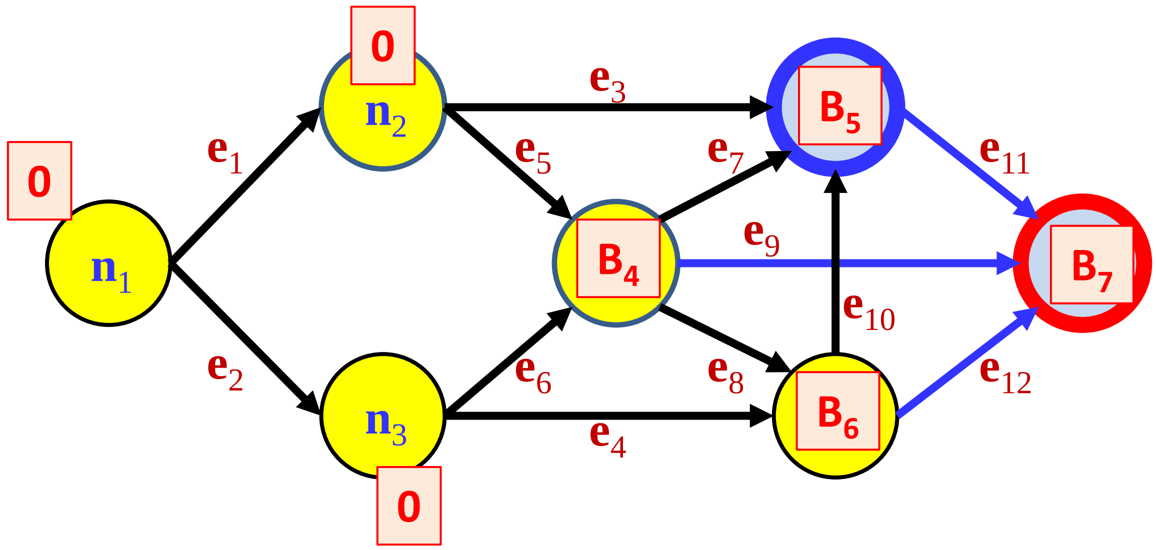
1. Mark sink nodes
2. Extend paths *backwards* from all sink nodes to all *parent* nodes
3. **Mark utilized sources and edges as "evaluated"**
   - Mark all utilized edges as "evaluated"
   - Mark all current source nodes as "evaluated"

Problem 3: The backward algorithm



1. Mark sink nodes
2. Extend paths *backwards* from all sink nodes to all *parent* nodes
3. Mark utilized sources and edges as "evaluated"
4. **Mark all *parent* nodes such that all outgoing edges are evaluated as "sink" nodes**

Problem 3: The backward algorithm



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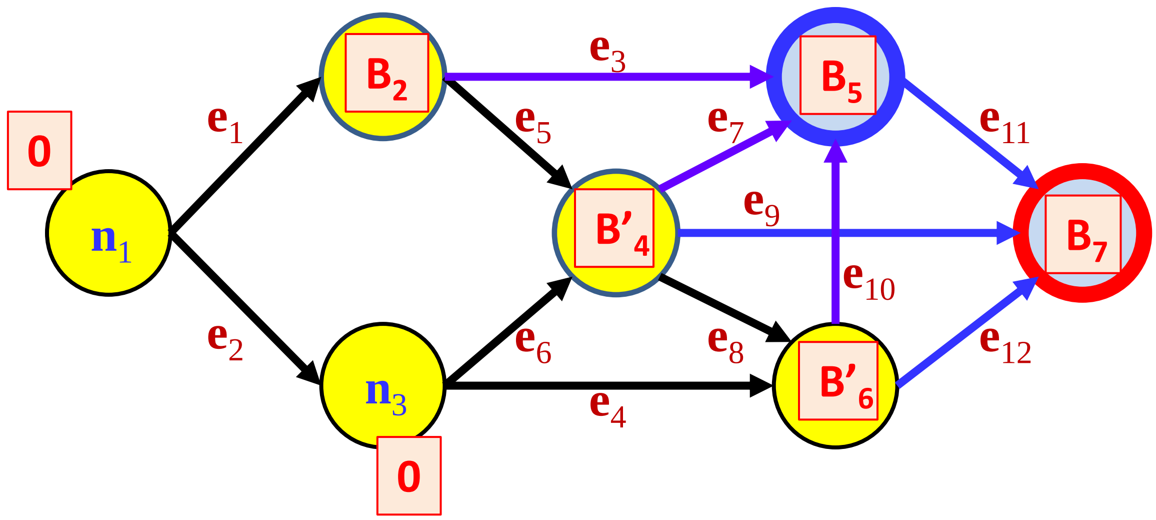

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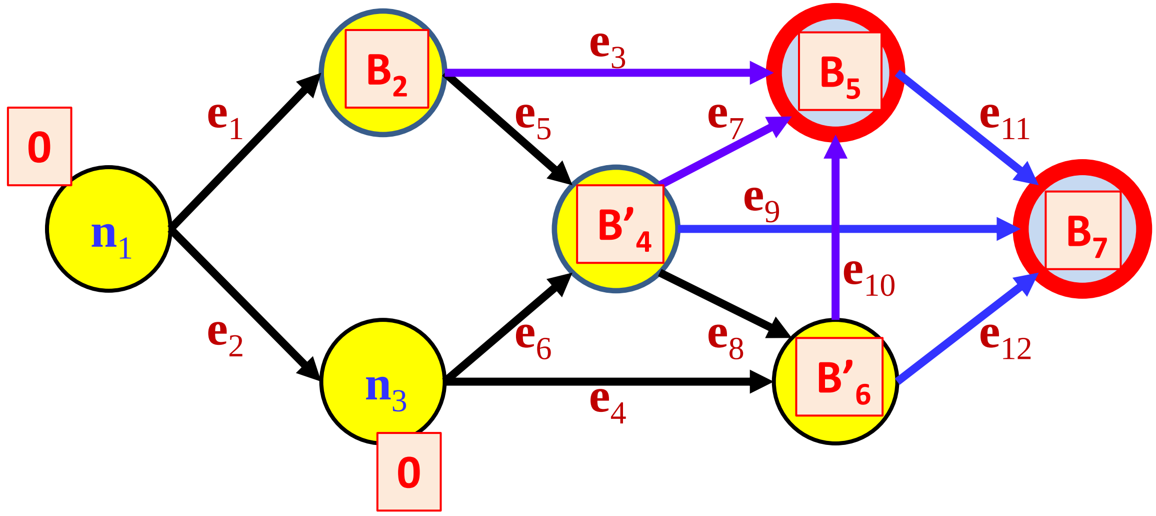

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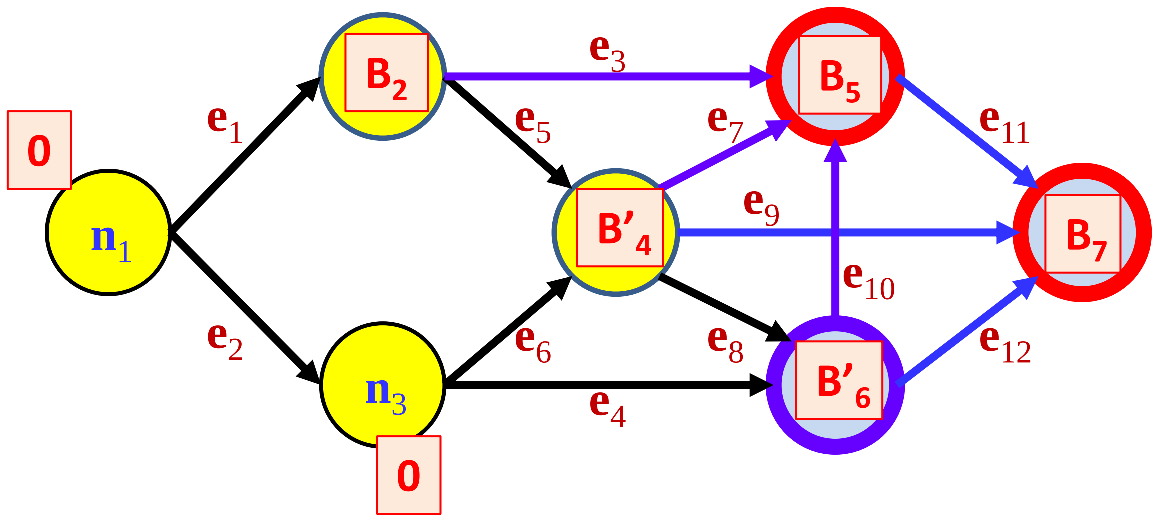

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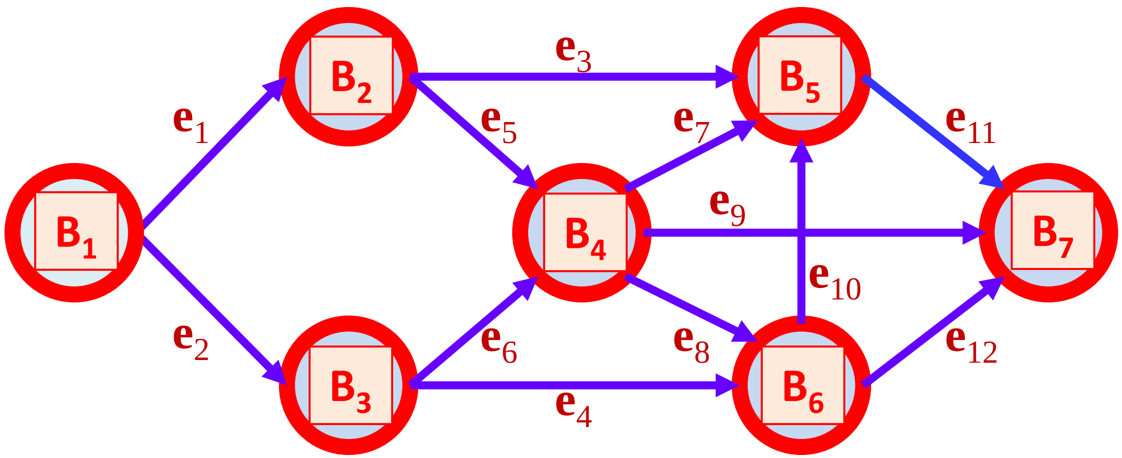
The Backward Algorithm

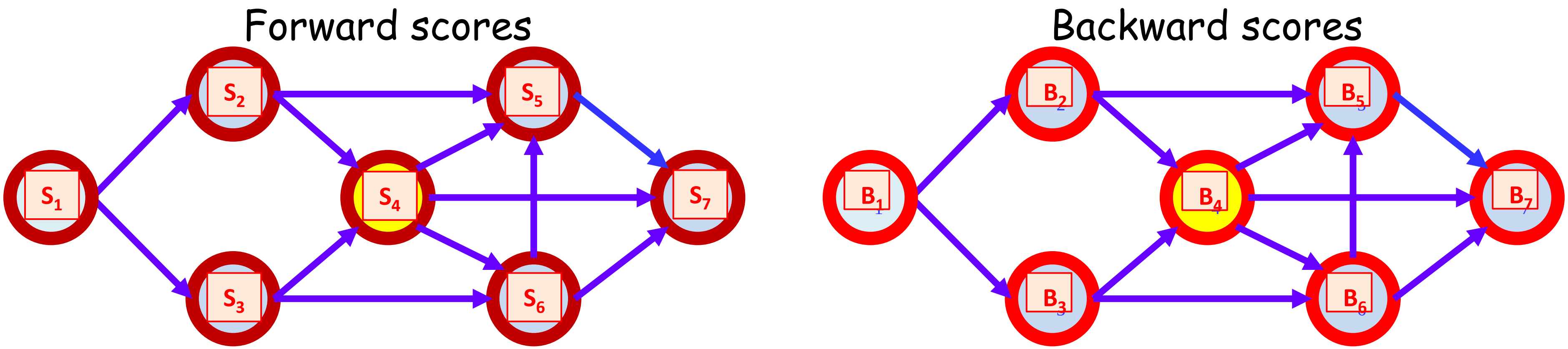


- **At termination**: The final score of any node is the total path score of *all* paths from that node to all *sink* nodes

The Forward-Backward Scores

- We can now compute the total score of all paths from all sources to all sinks that pass through a specific node

The Forward Backward Algorithm



- Total cost of all paths through node 4 = $S_4 * B_4 / n_4$
  - In general, for any node *i*, total cost = $S_i * B_i / n_i$
  - Assuming probability-based combination

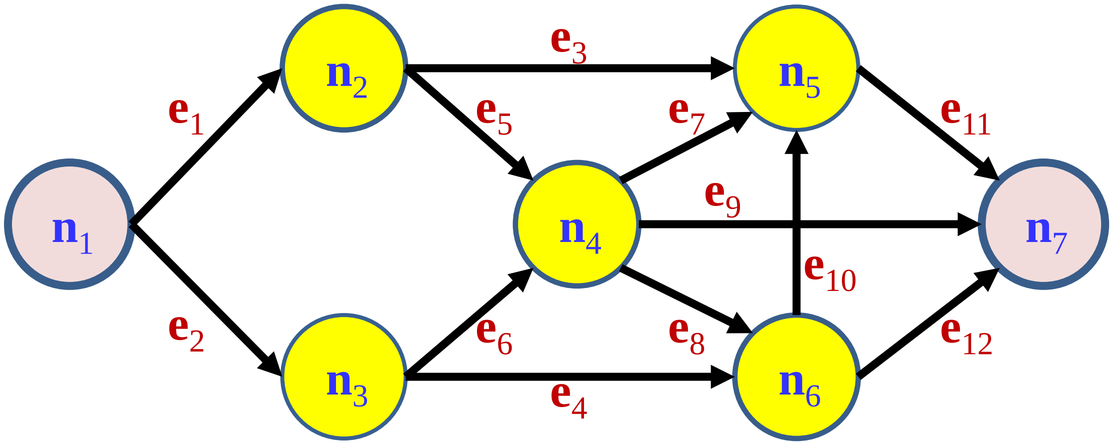
- Forward score * Backward score / node score
  - S_i = forward score, B_i = backward score; n_i = node score
  - Must divide out n_i since it is included in both forward and backward scores
  - Division eliminates duplication

YET another graph problem: N-best

- We have seen how to find the score the shortest path between a source and a sink node
  - And, consequently, the shortest path itself

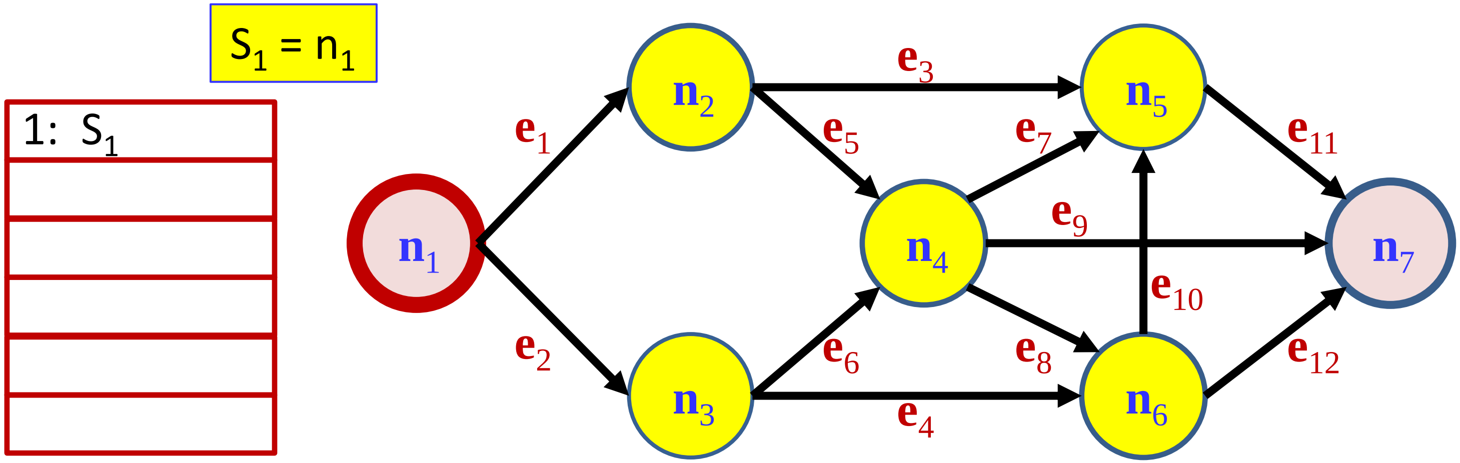
- But what is the length of the *second* shortest path?
  - Or the N-th shortest path
  - *What are these paths?*

The *n-shortest paths* problem



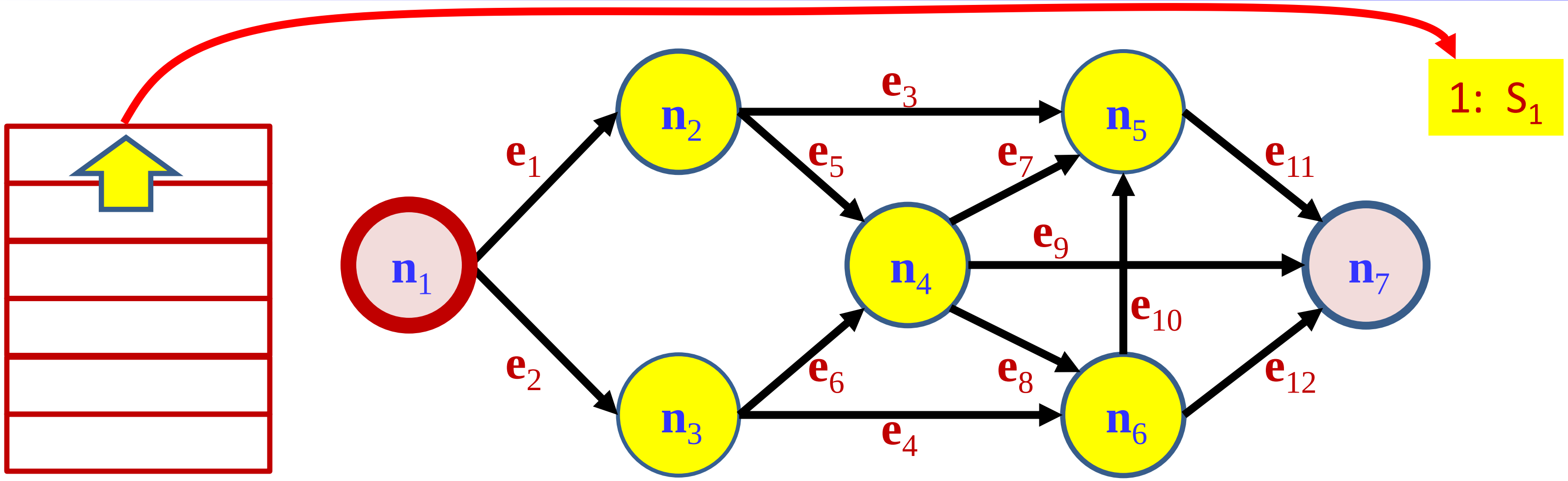
- What are the *N* shortest paths between the source and sink nodes?
  - The "Stack" decoder
  - The "A*" algorithm

The *stack decoder*



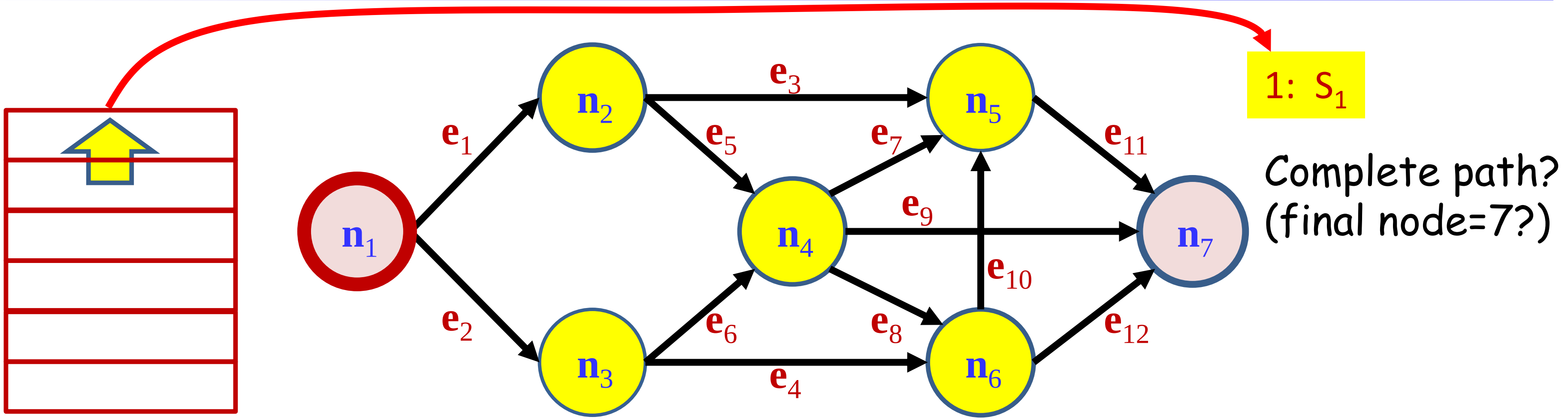
- Begin at the source
  - The total cost of the path thus far is simply n_1
    - $S_1 = n_1$
  - Push "1:S_1" into a "stack"
    - "1" identifies the path, S_1 is its score

The *stack decoder*



1. Pop current shortest partial path from stack

The *stack decoder*



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   else: go to 3

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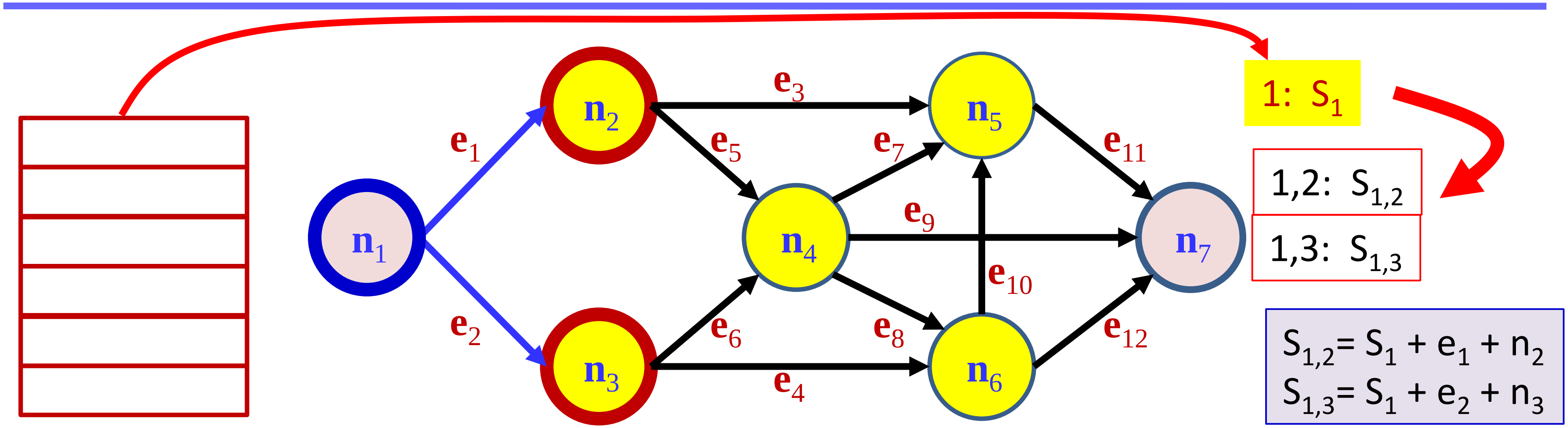

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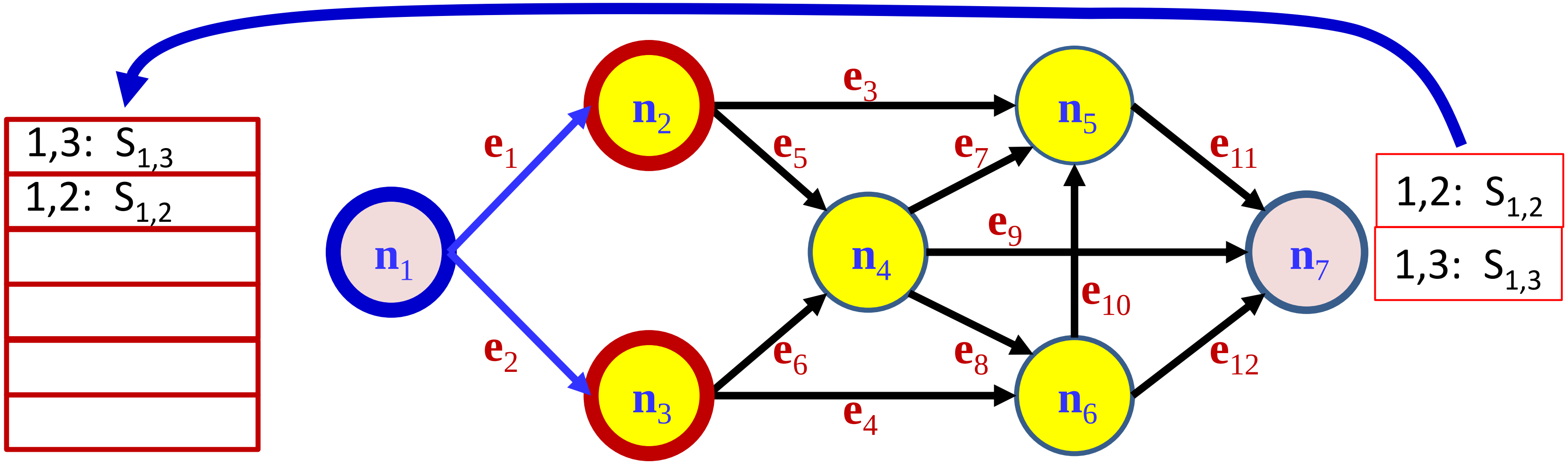

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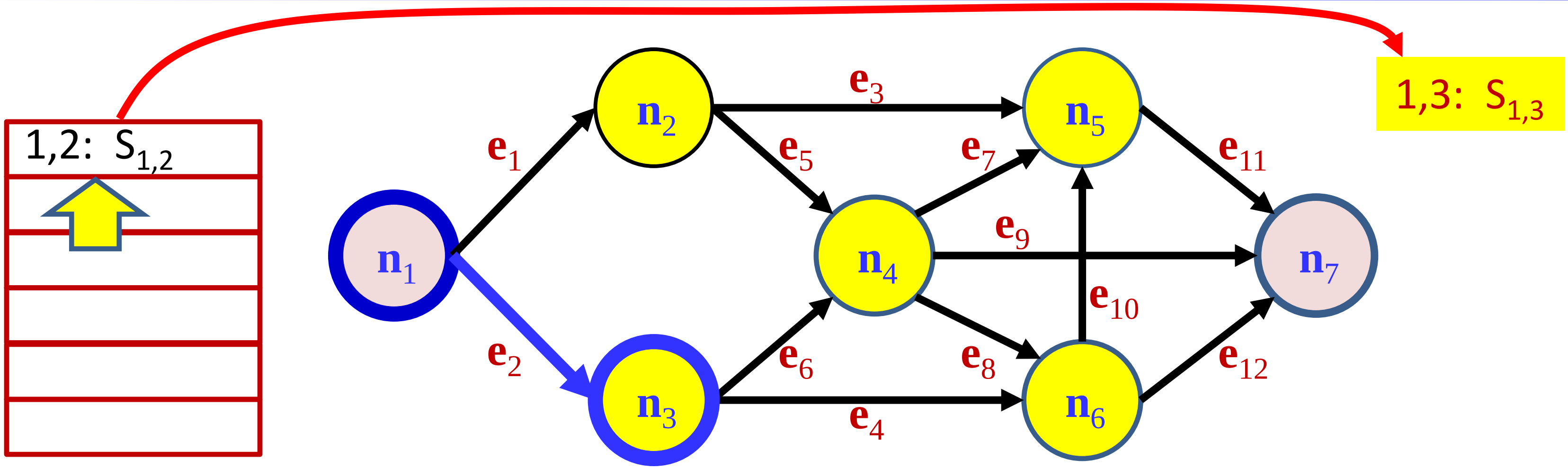

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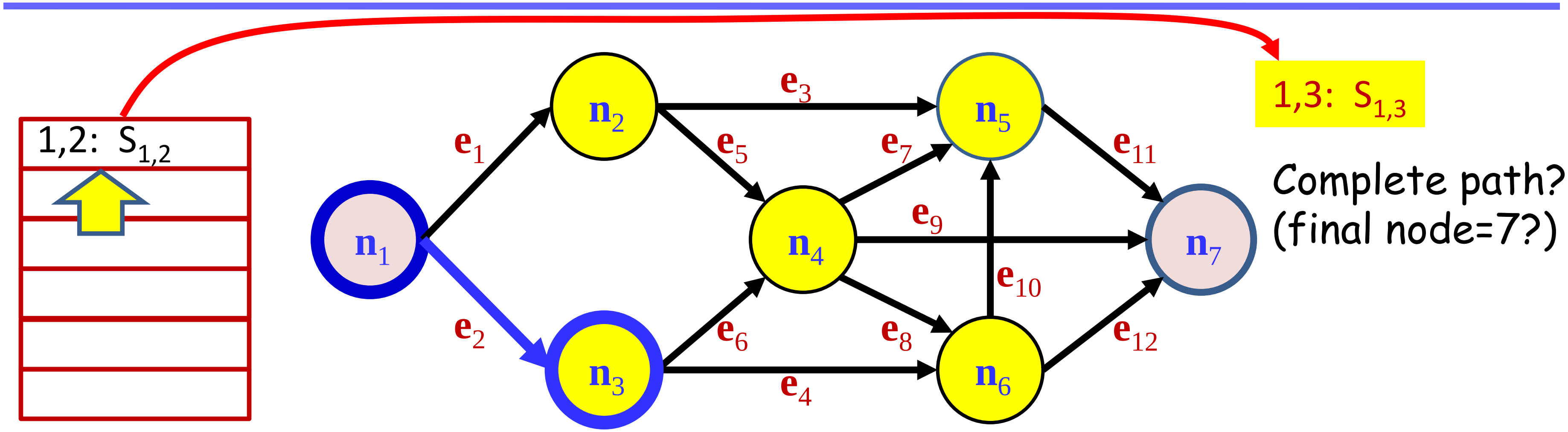

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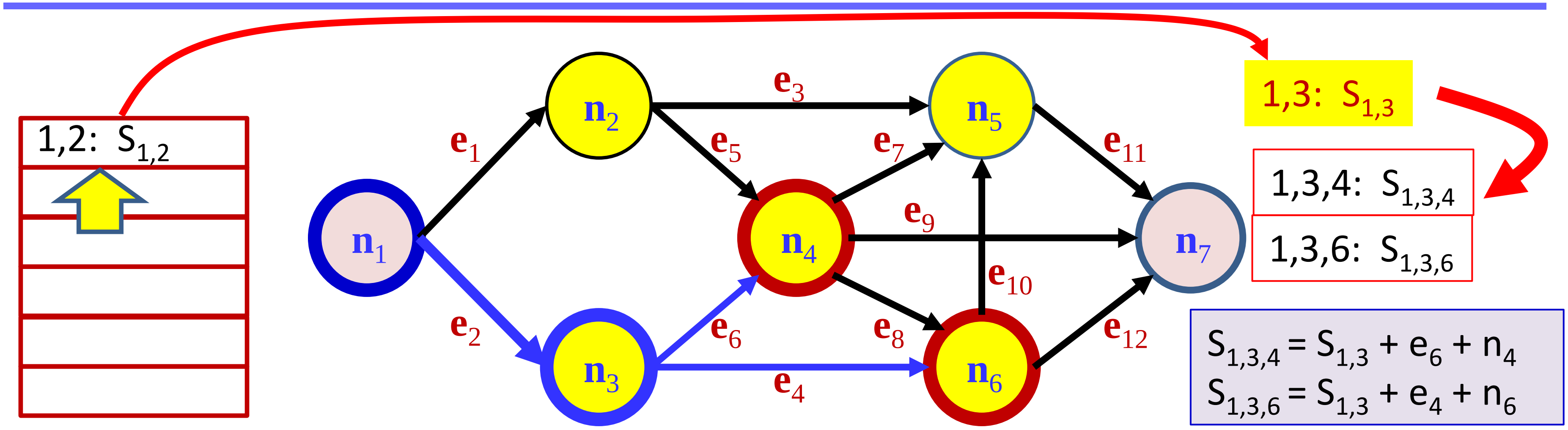

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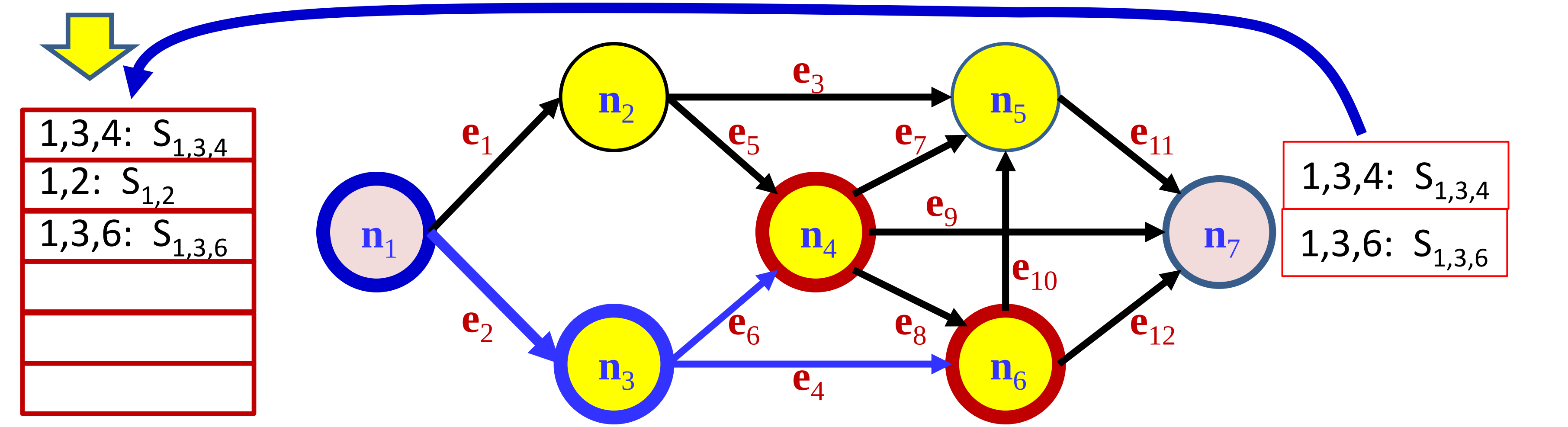

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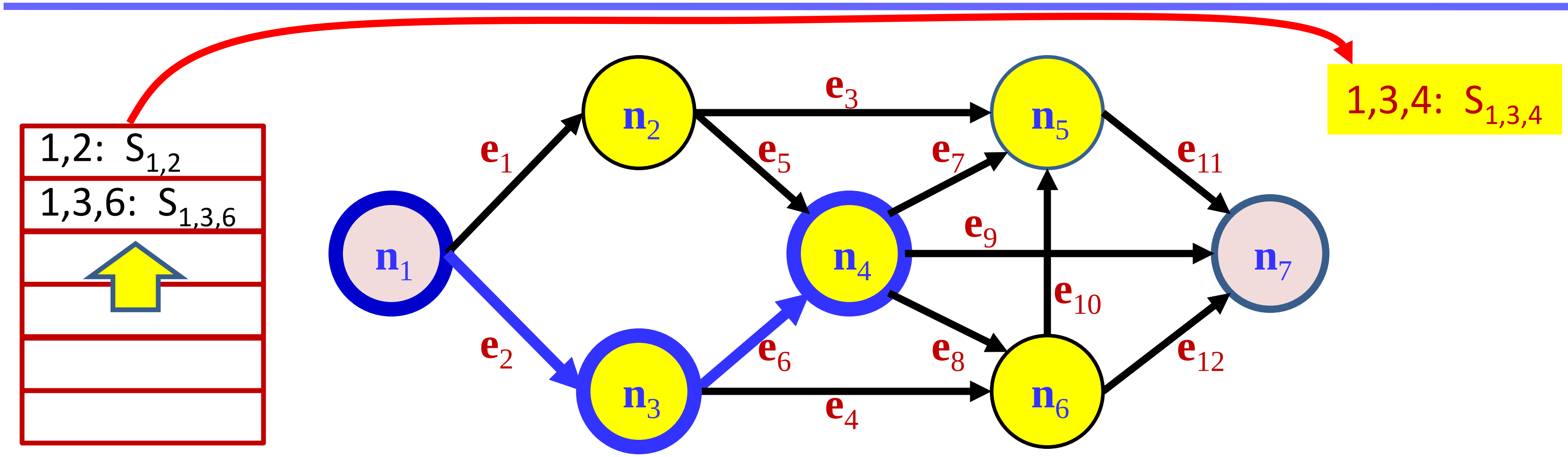

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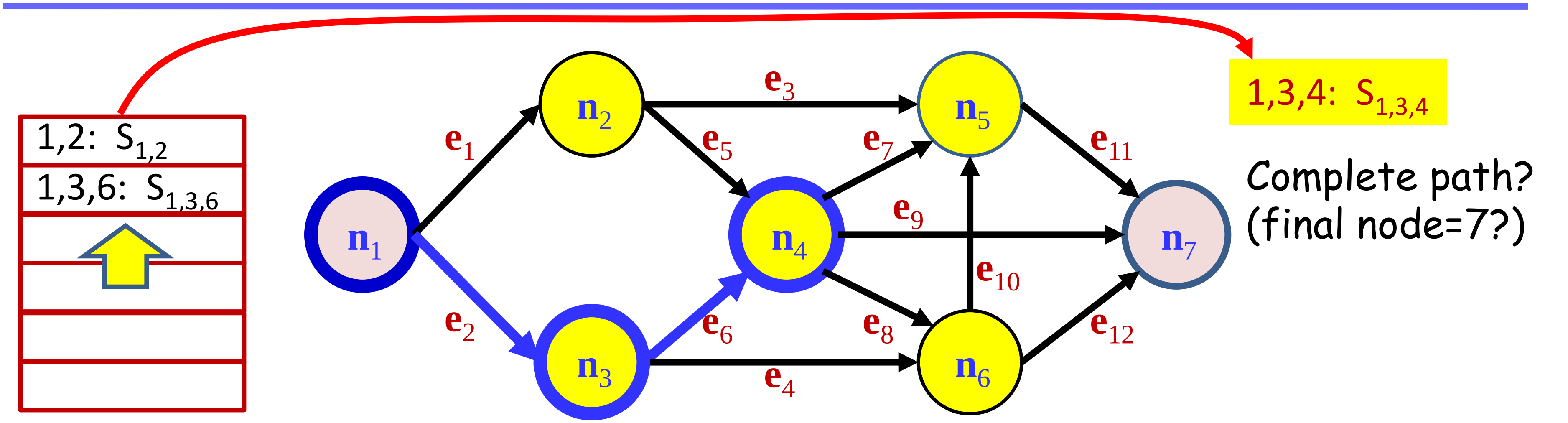

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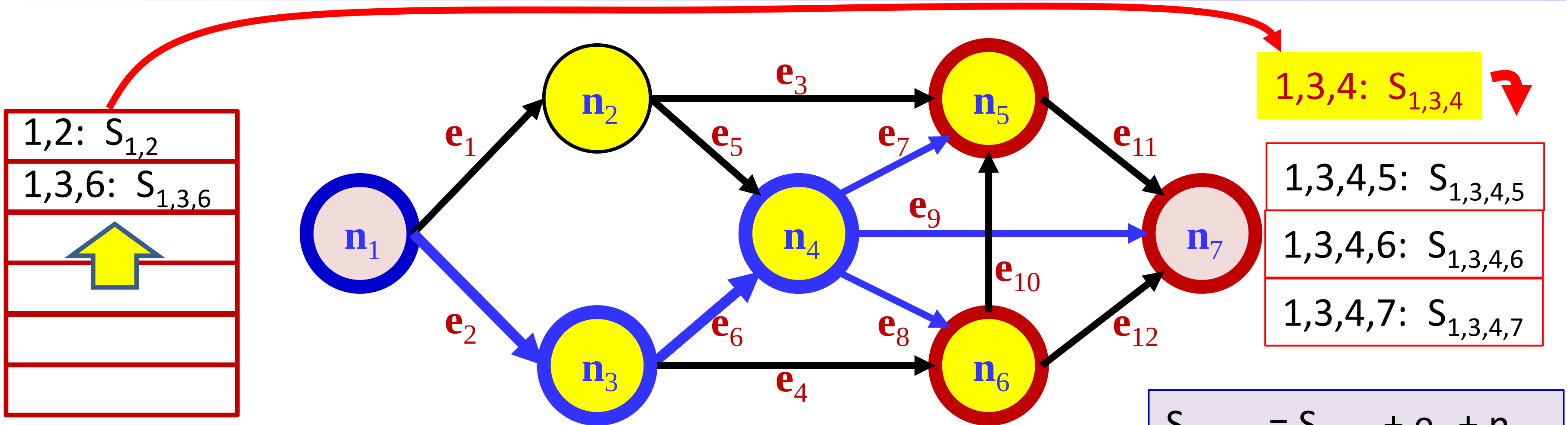

$S_{1,3,4,5} = S_{1,3,4} + e_7 + n_5$
$S_{1,3,4,6} = S_{1,3,4} + e_8 + n_6$
$S_{1,3,4,7} = S_{1,3,4} + e_9 + n_7$

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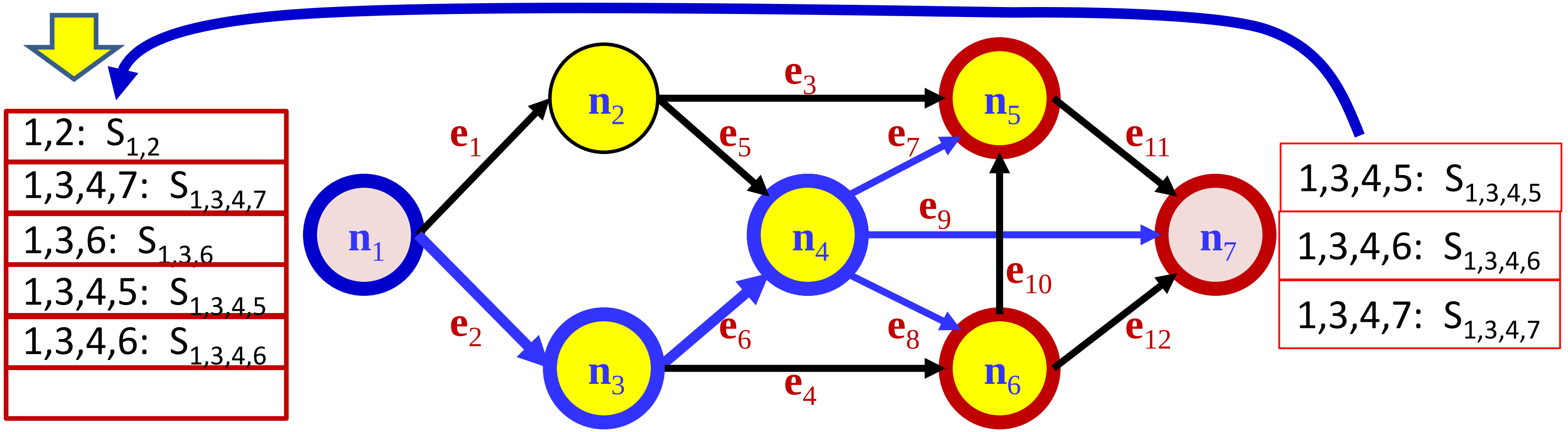

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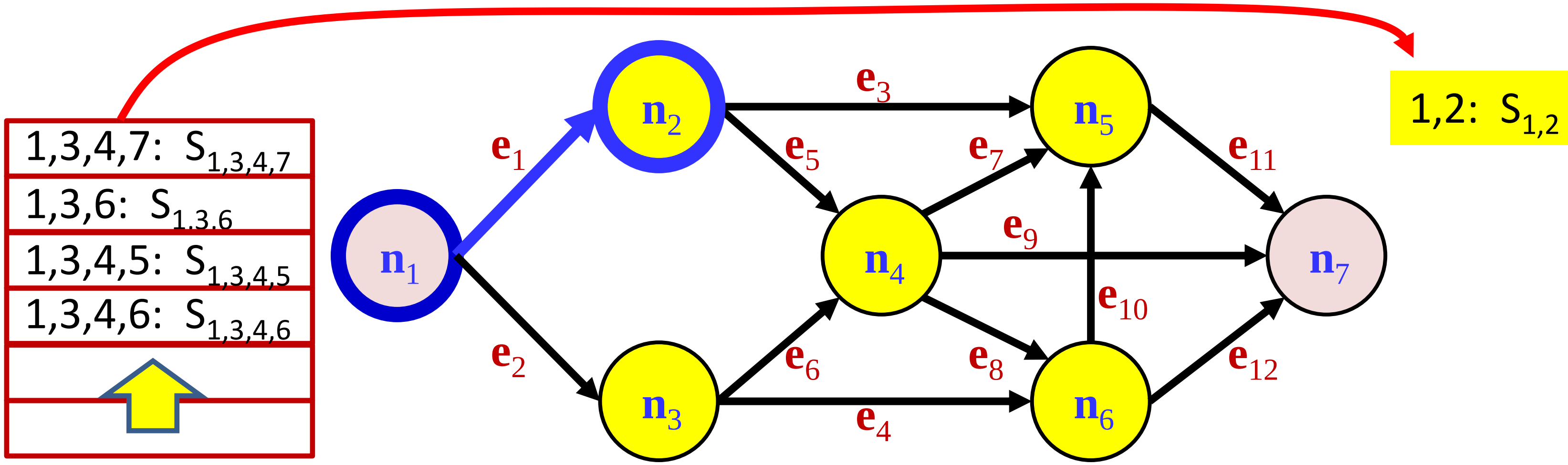

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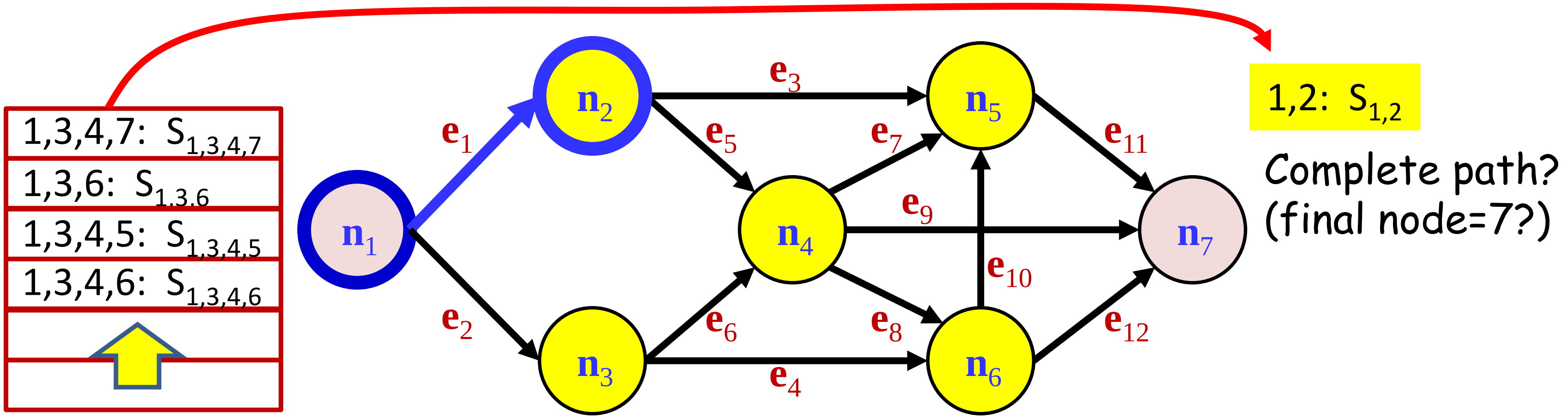

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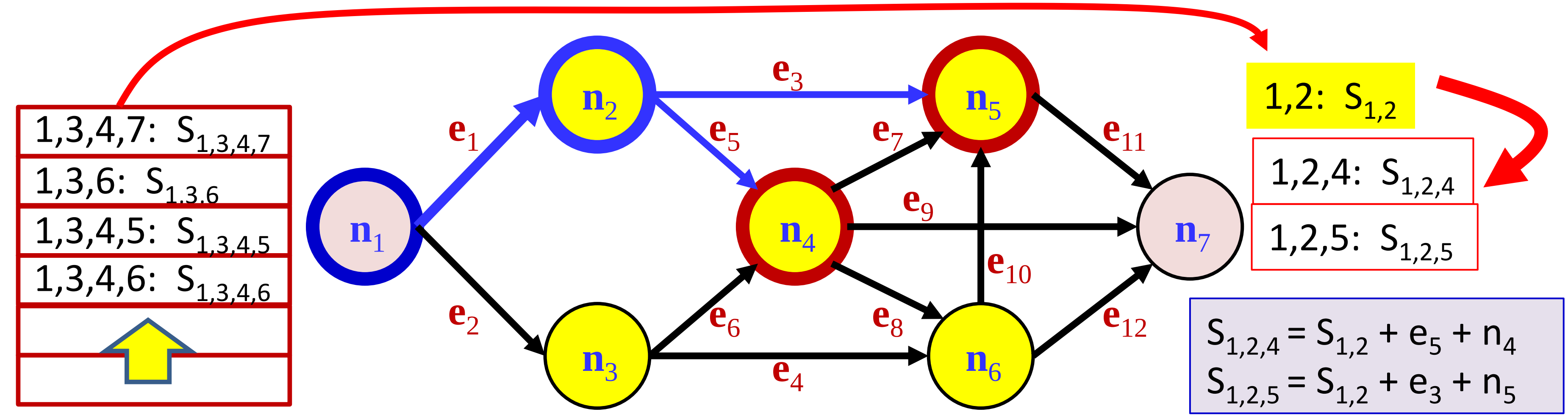

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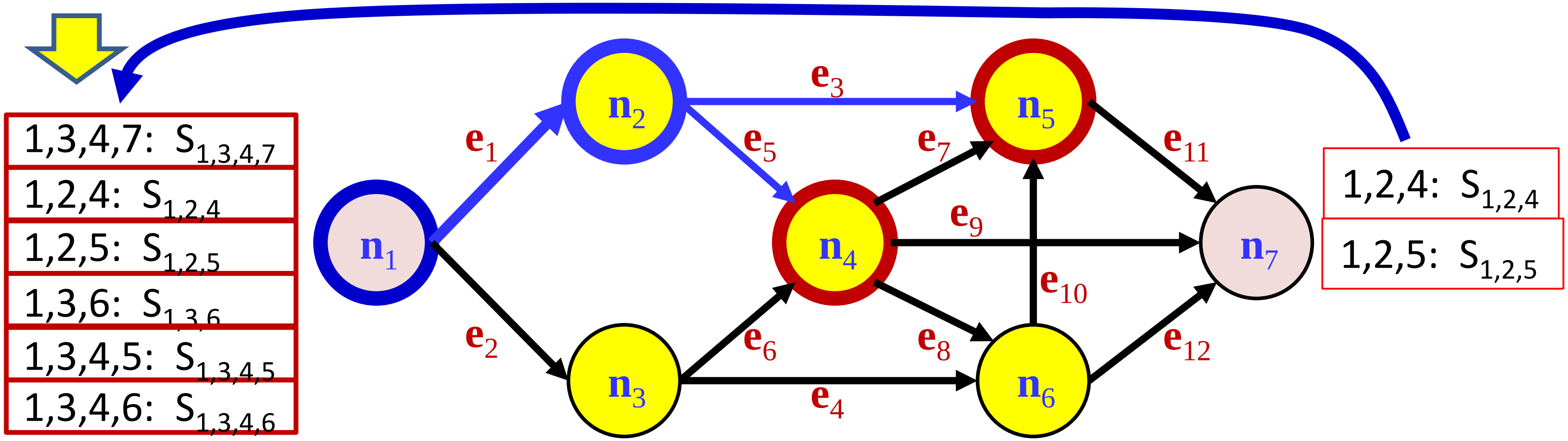

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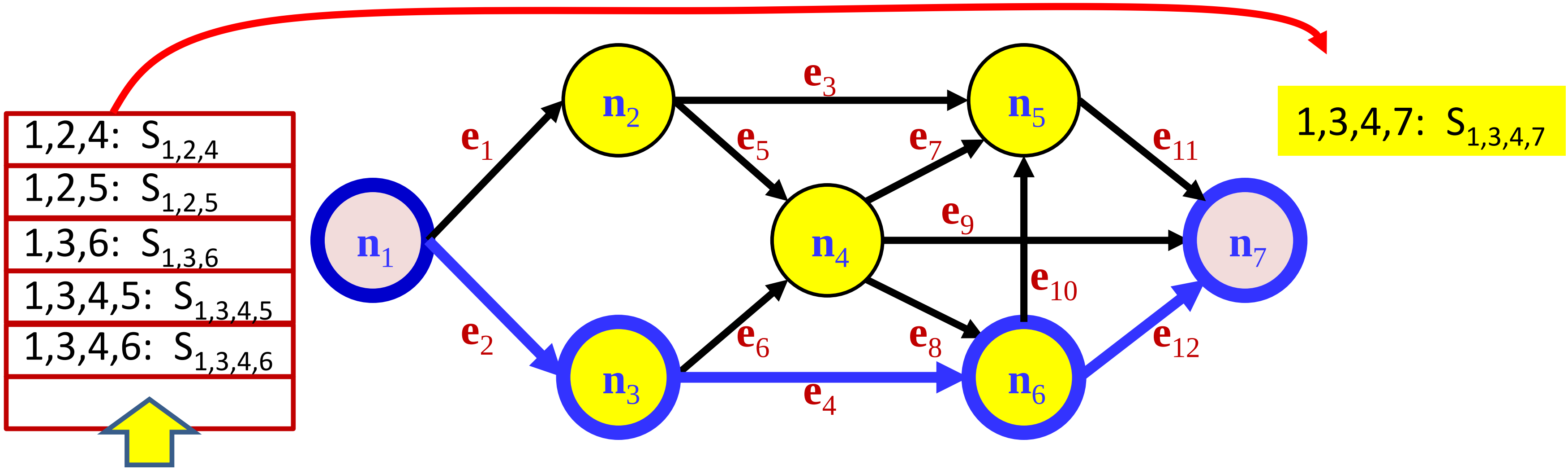

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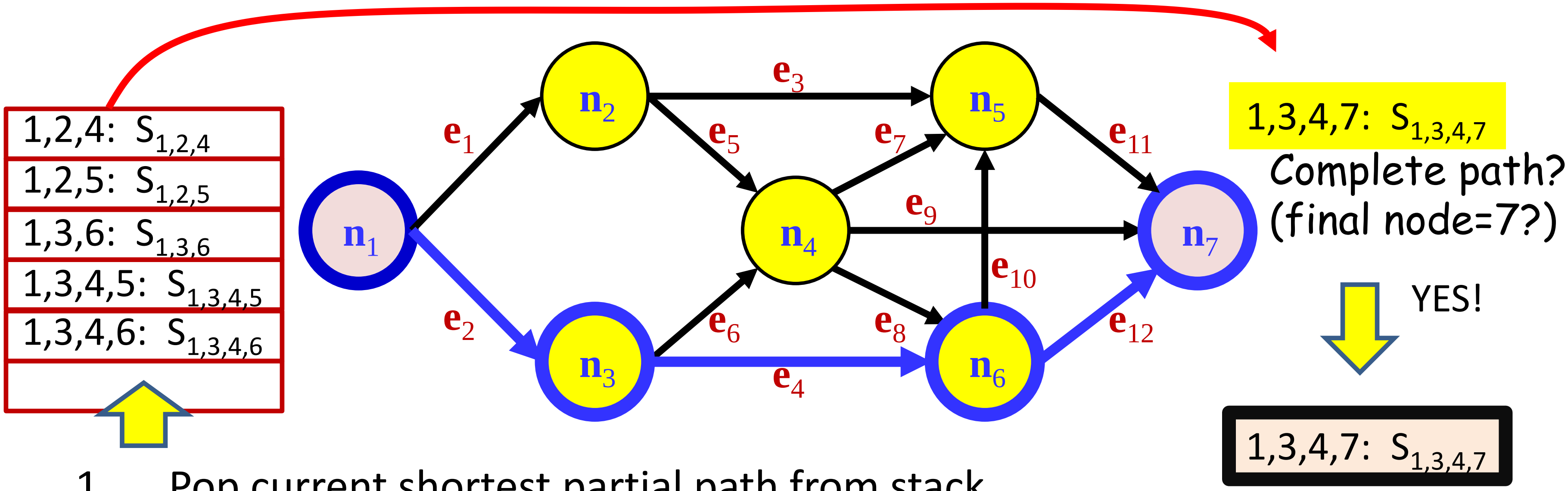

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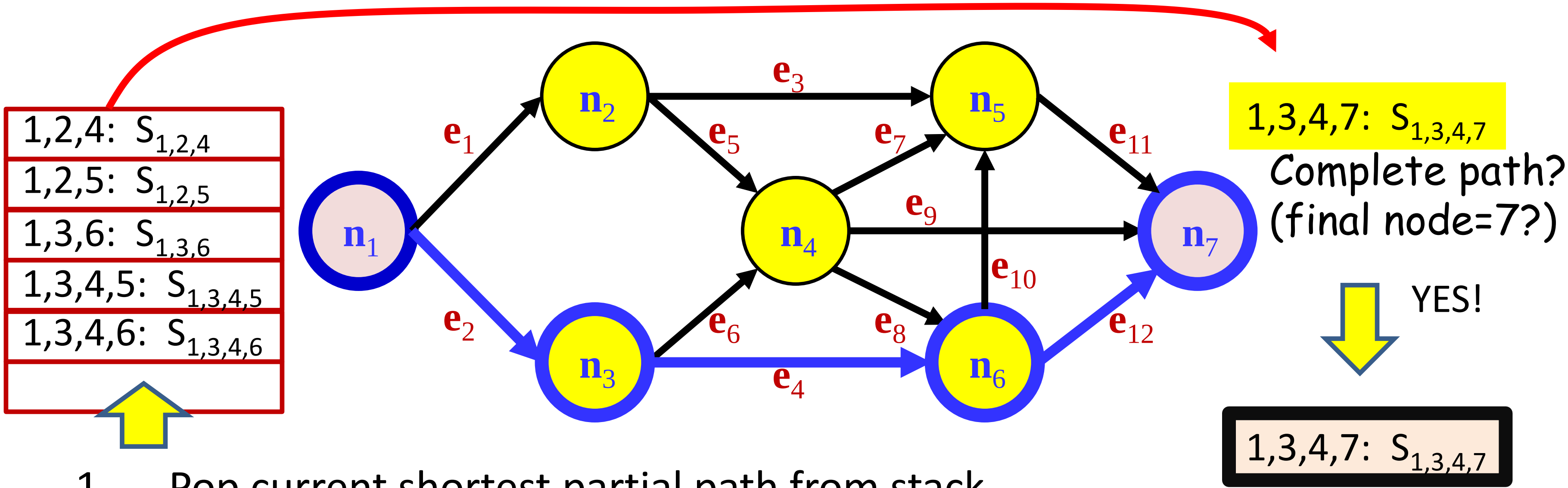

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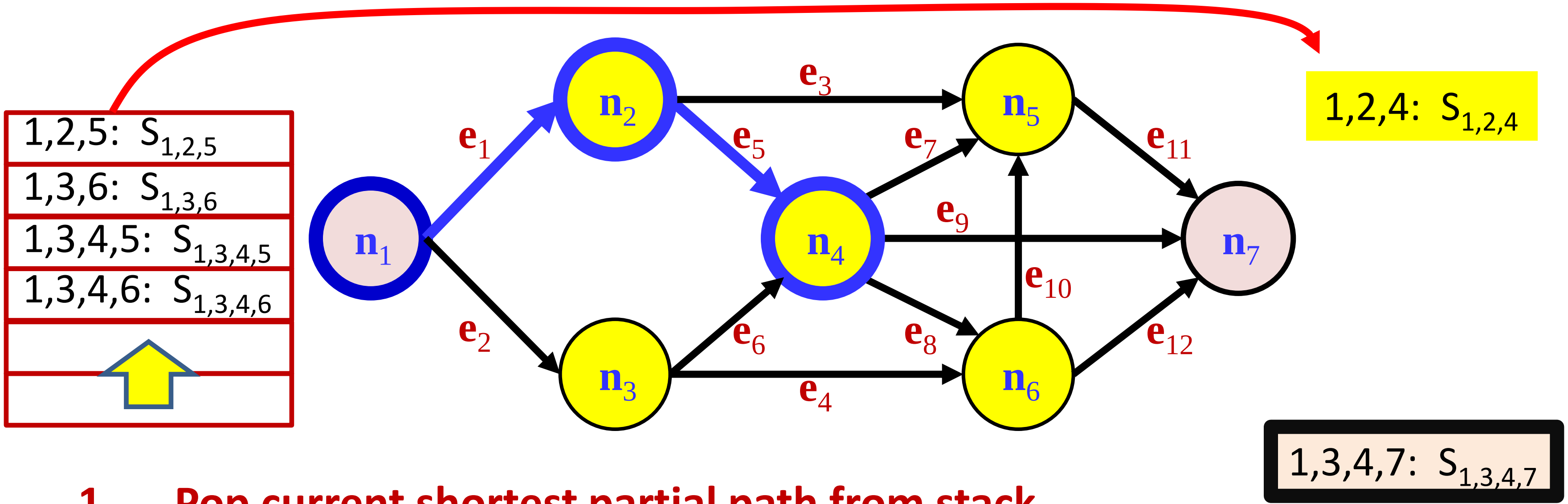

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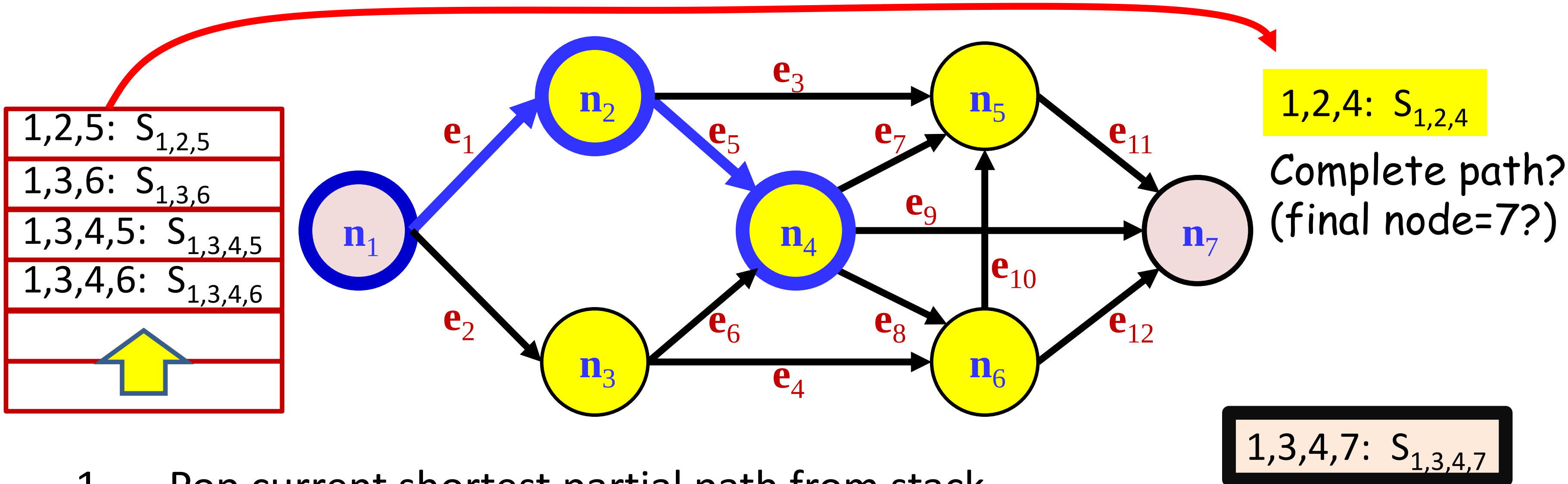

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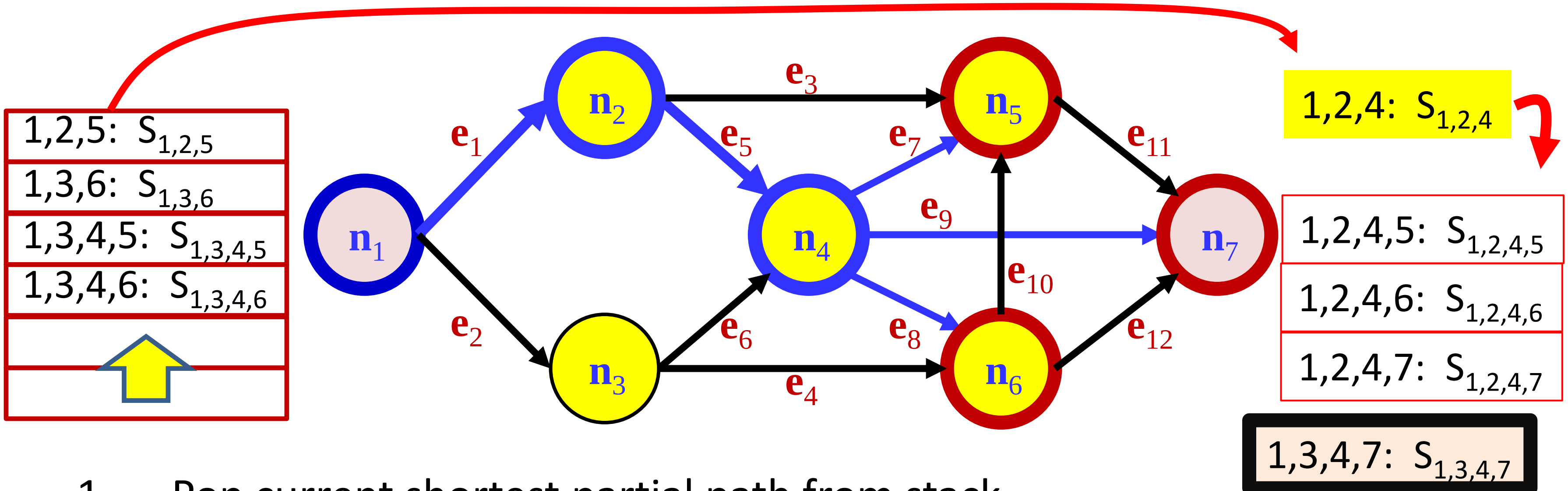

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The stack decoder

- The algorithm continues until the desired number (N) of paths are output

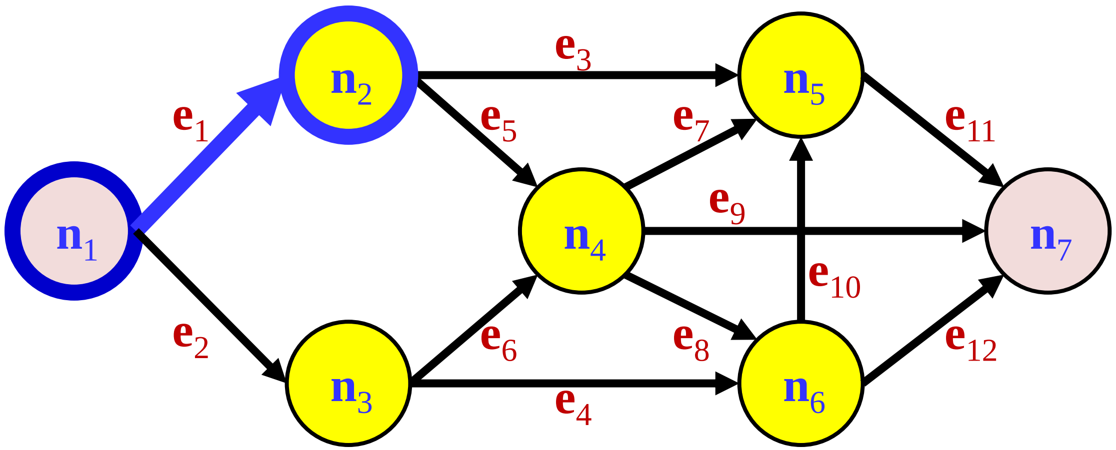
- The process *guarantees* that these are the N shortest (lowest-cost) paths through the graph

- Computational complexity: Upper bound = N!
  - In practice, much much smaller

- Still, it has a problem:
  - Frequently its very slow
  - Why?...

Problem with the stack decoder

1,2: $S_{1,2}$
1,3,4,7: $S_{1,3,4,7}$
1,3,6: $S_{1,3,6}$
1,3,4,5: $S_{1,3,4,5}$
1,3,4,6: $S_{1,3,4,6}$



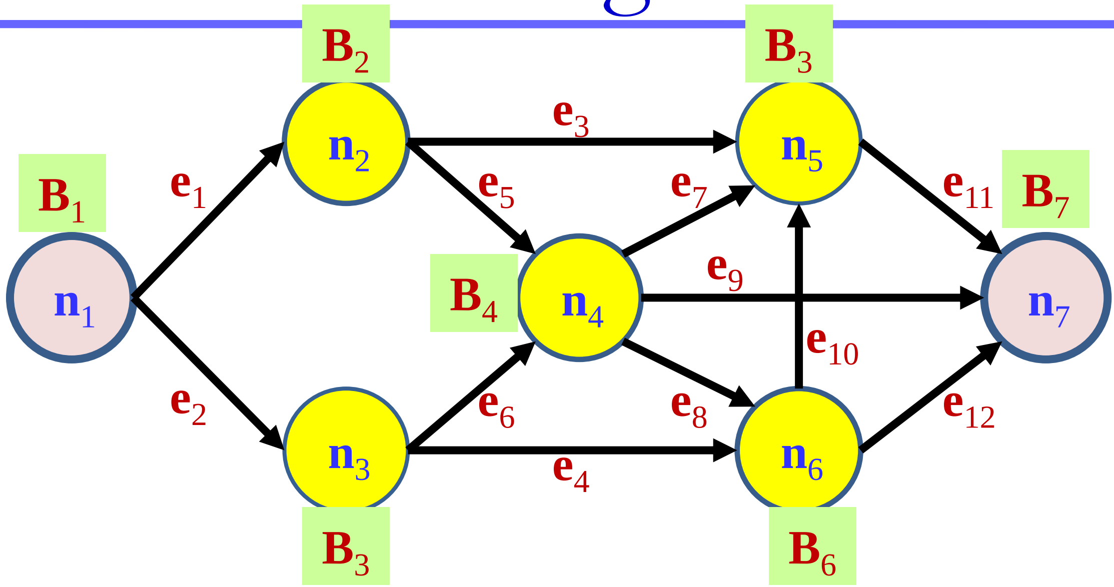
- The top of the stack is often dominated by paths that are close to the source
  - They tend to have lower costs than paths that are closer to the sink
- The algorithm will preferentially pop these
  - Effectively spending most of the time expanding shallow paths, instead of exploring the more promising deeper ones

The A* Algorithm

- Solution: *Predict the future*
  - Replace every score $S_{*,b}$ with $S_{*,b} + B_{b,sink}$
  - $S_{*,b}$ is the score of a path ending at node b
  - $B_{b,sink}$ is a *guess* of the lowest cost score from b to the sink
- Note that setting $B_{b,sink} = 0$ results in the conventional stack decoder

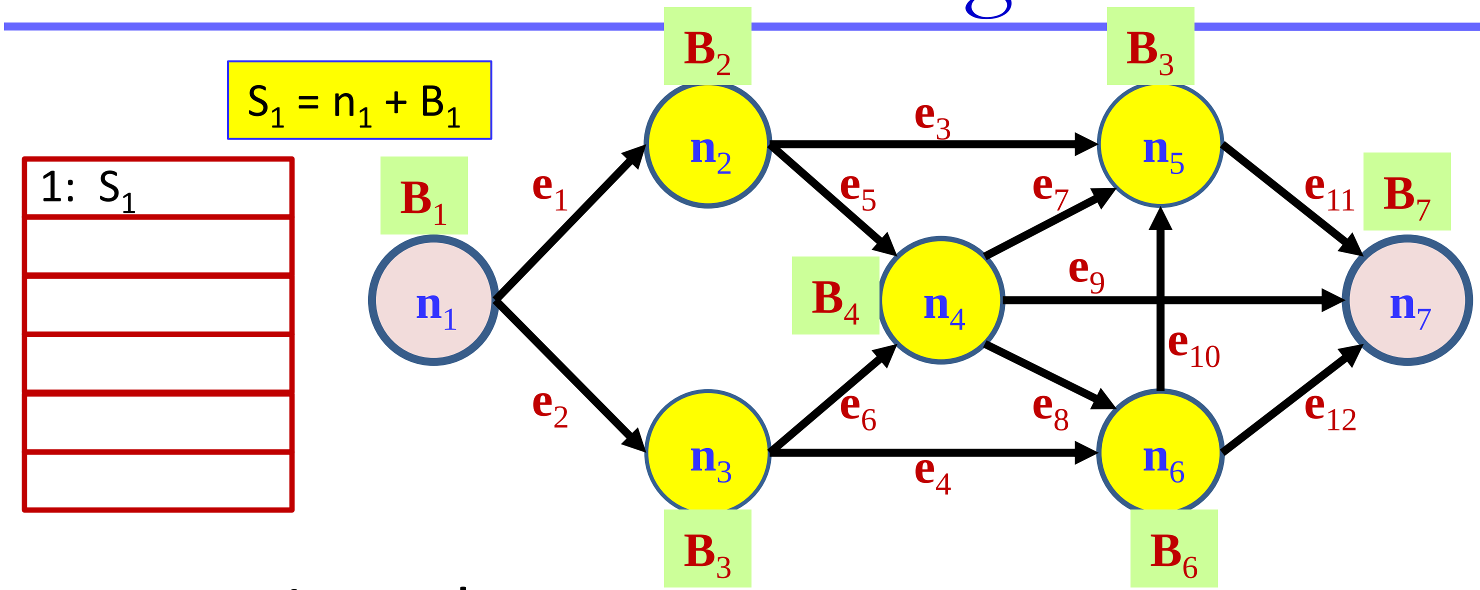
- **Guarantee:** If $B_{b,sink}$ is a **true *lower bound*** on the the best path score from b to the sink, the A* algorithm returns the correct results
  - I.e. the same result as the stack decoder
  - Only much much faster

The A^* algorithm



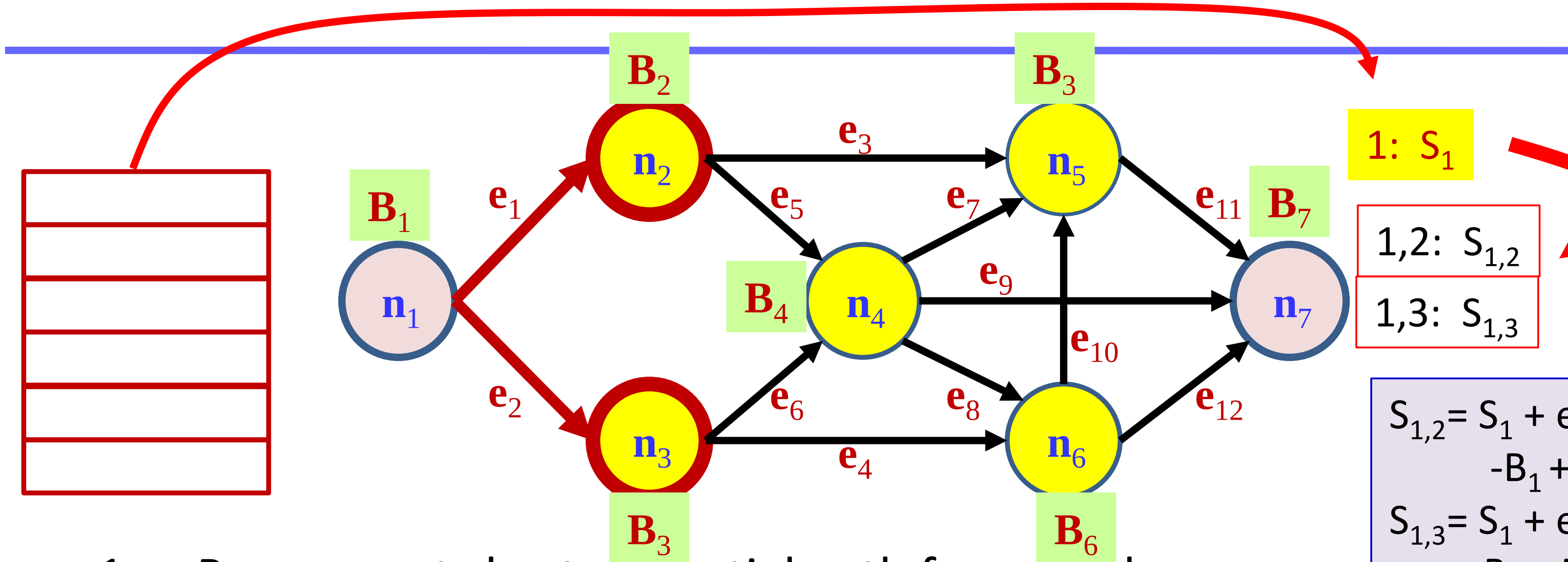
- First: Compute the best path cost from each node to the sink node
  - Can be computed using Dijkstra's algorithm

The A^* algorithm



- Begin at the source
  - **The total cost of the path is the forward path cost to the node PLUS the (guessed) best path cost to the sink**
    - **$S_1 = n_1 + B_1$**
  - Push "1:S_1" into a "stack"
    - "1" identifies the path, S_1 is its score

A^*



$S_{1,2} = S_1 + e_1 + n_2 - B_1 + B_2$

$S_{1,3} = S_1 + e_2 + n_3 - B_1 + B_3$

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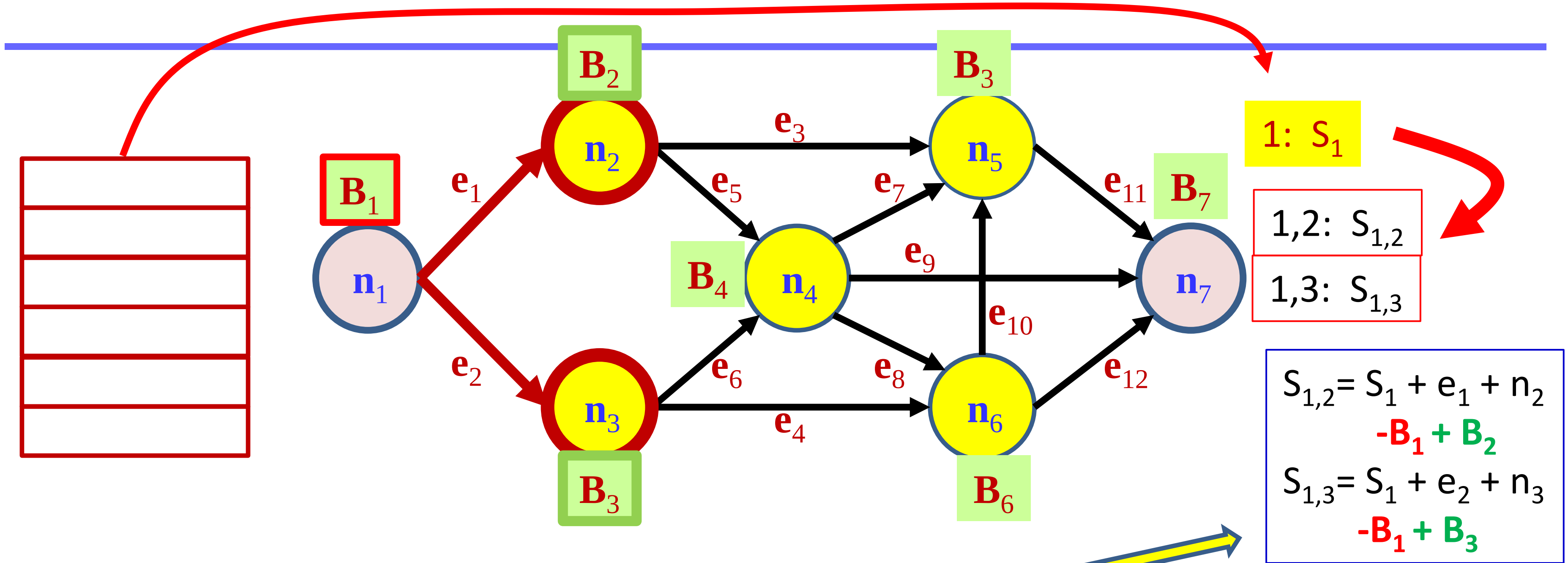
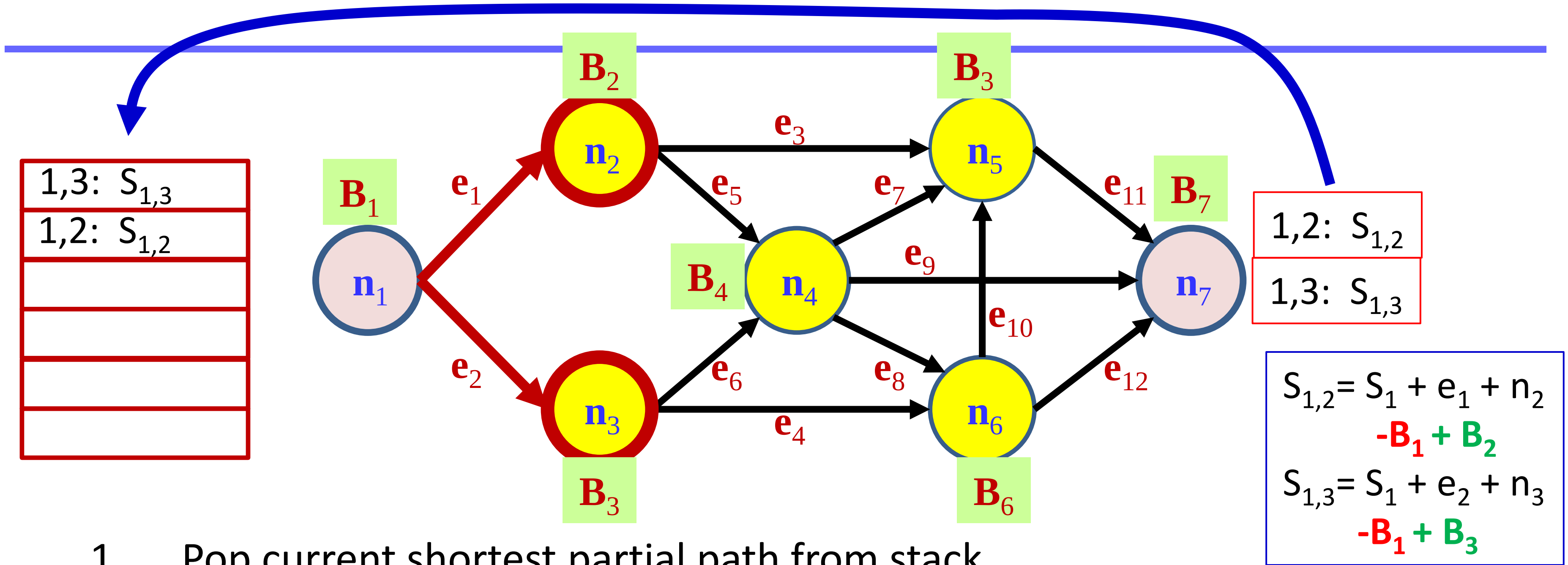
A^*



- NOTE: Modified Score Computation
- Subtract B_1
  - Subtract ***previous*** (best guess of) lowest cost of remaining path to sink
- Add B_{node}
  - Add ***current*** (best guess of) of lowest cost of remaining path from current node to sink

A^*



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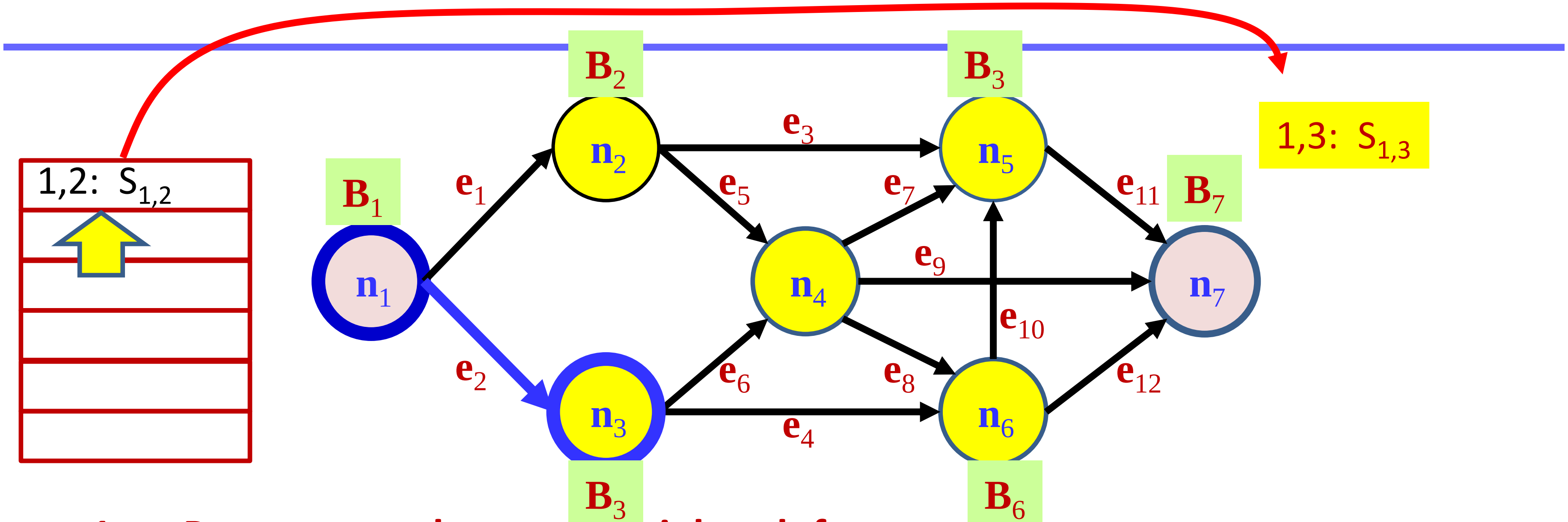
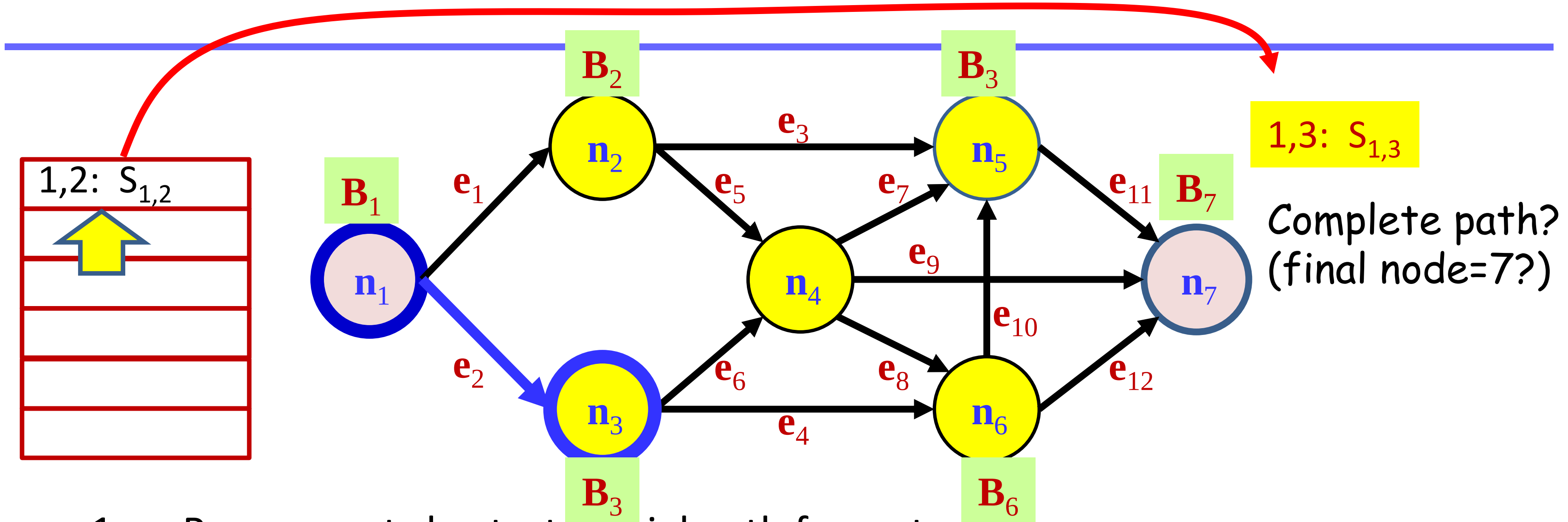
A^*



1. **Pop current shortest partial path from stack**
2. If : final node of partial path is sink node, output it
   - If desired number (N) of outputs obtained : terminate
     else: return to 1.

   else: go to 3
3. Extend partial path by expanding all edges of final node on path
4. Push all extended paths into stack
   - Arrange stack by increasing cost: lowest cost path on top
5. **Return to 1.**

$\mathcal{A}^*$



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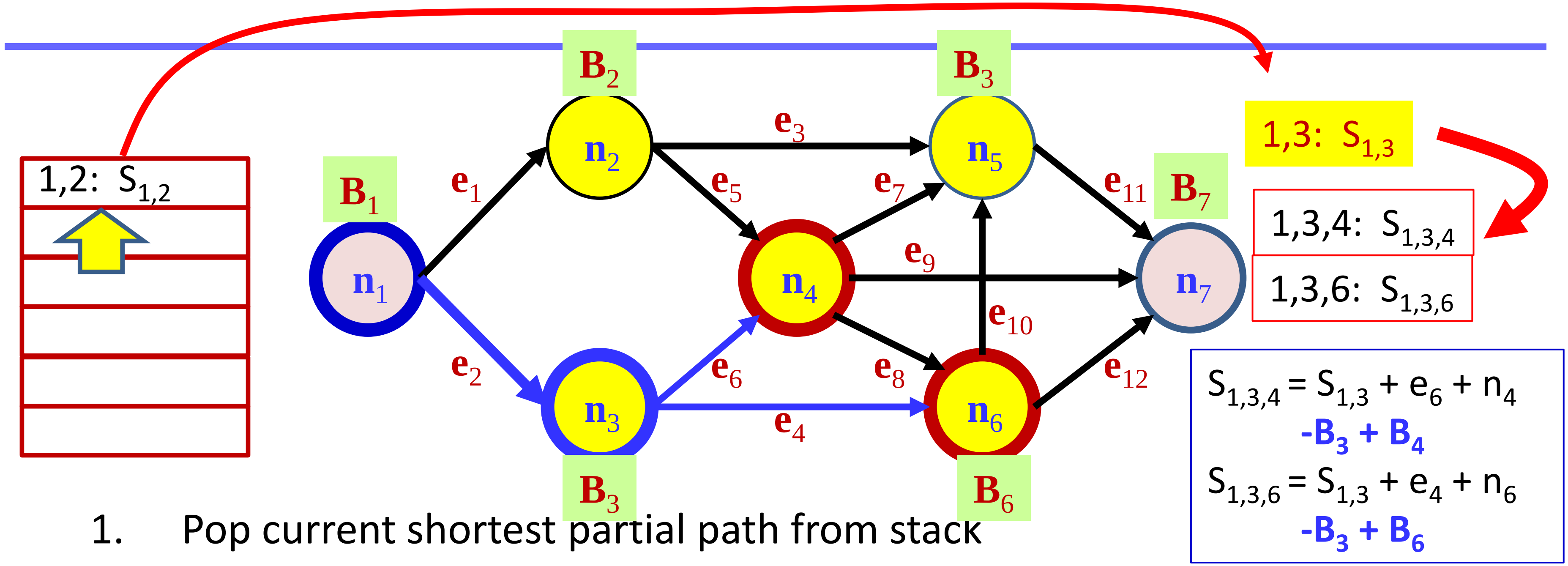

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The A* Algorithm

- The A* proceeds as the stack decoder does, with the modification that predicted future scores are always incorporated

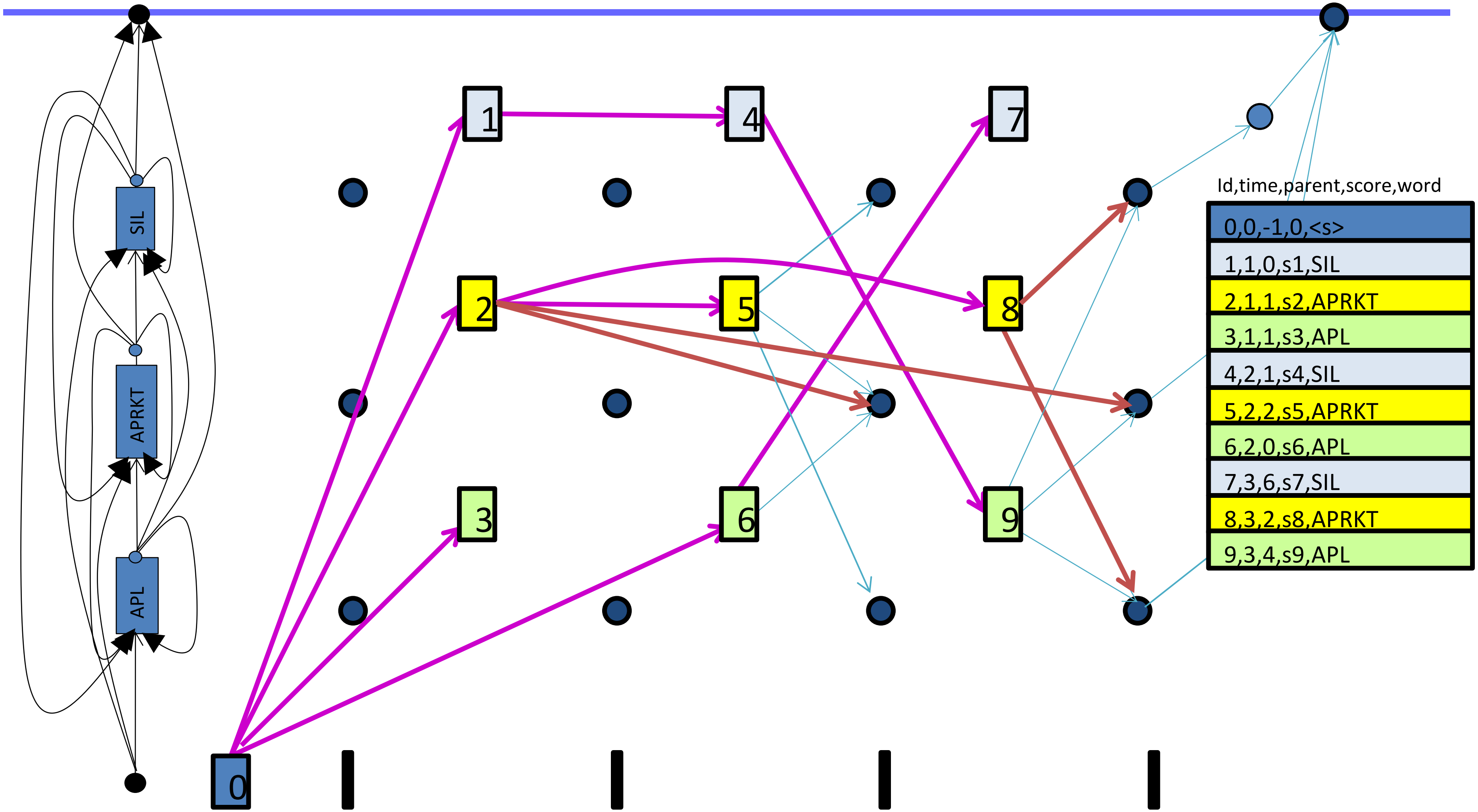
- Caveats:
  - For the predicted future score do *not* include the score of the first node
    - To ensure that the node score is not included twice in any path score
    - E.g. B_3 must not include n_3
    - This can be done by explicitly subtracting out n_3 from the best path score computed by Dijkstra's algorithm

Returning to ASR

- We now apply what we have learned to address some problems in speech recognition

- N-best generation
- Rescoring
- Confidence estimation

The Backpointer Table is a Tree



- Note LM probabilities now

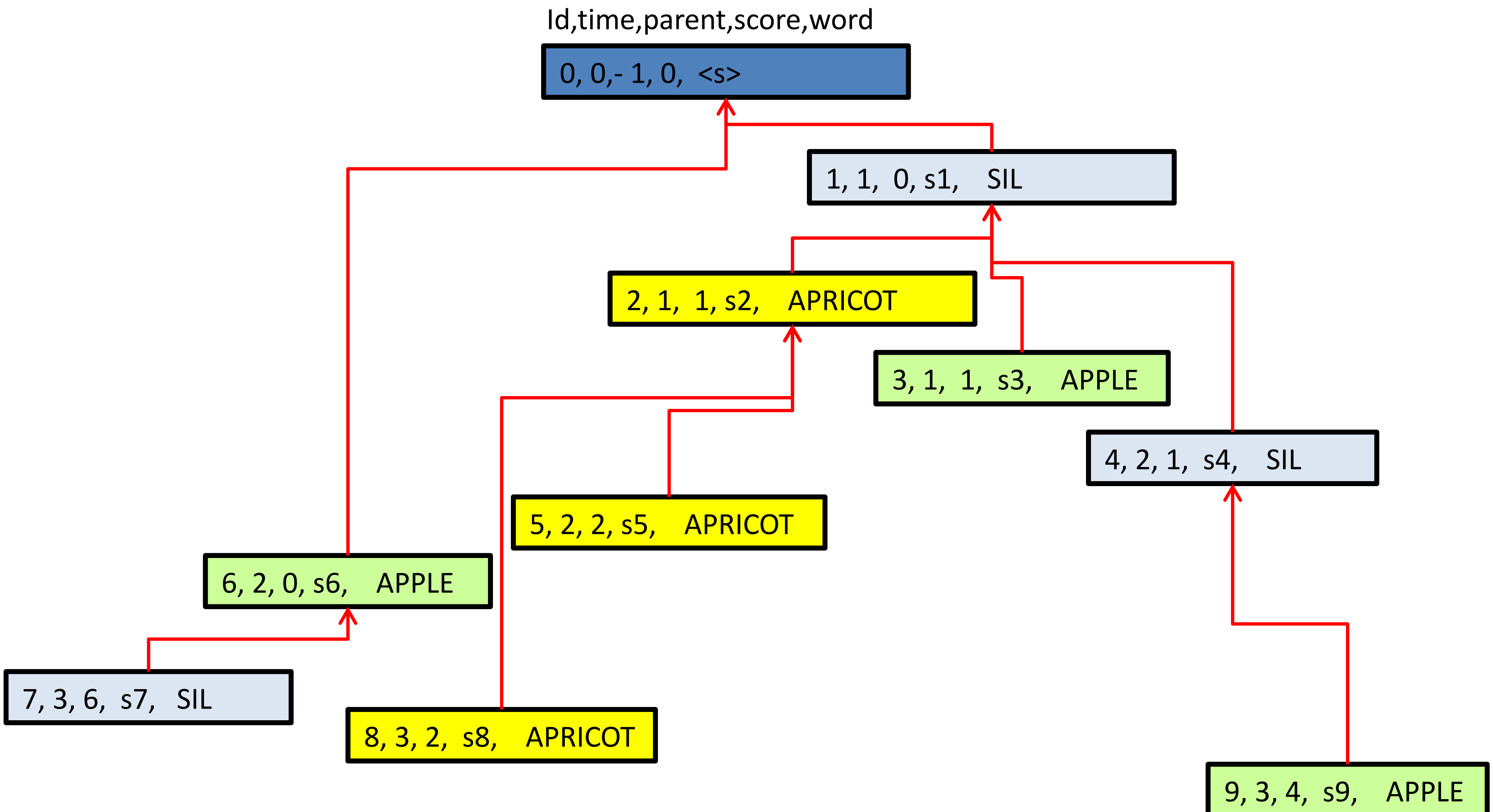
The BackPointer Table is a TREE

- The backpointer table is a tree

Id	time	parent	score	word
0	0	- 1	0	<s>
1	1	0	s1	SIL
2	1	1	s2	APRICOT
3	1	1	s3	APPLE
4	2	1	s4	SIL
5	2	2	s5	APRICOT
6	2	0	s6	APPLE
7	3	6	s7	SIL
8	3	2	s8	APRICOT
9	3	4	s9	APPLE

The BackPointer Table is a TREE

- The backpointer table is a tree



The Backpointer Table

- Each entry in the BP table has:
  - An *end* time
  - An implicit *start* time
    - End time of parent +1
  - A *word* identity
  - A node score
    - Total score to node – total score to parent
    - Node score may be further separated into
      - Acoustic score
      - Language score
      - Must keep track of acoustic and language model scores separately for this

Id,time,parent,score,word

0	0	- 1	0	<s>
1	1	0	s1	SIL
2	1	1	s2	APRICOT
3	1	1	s3	APPLE
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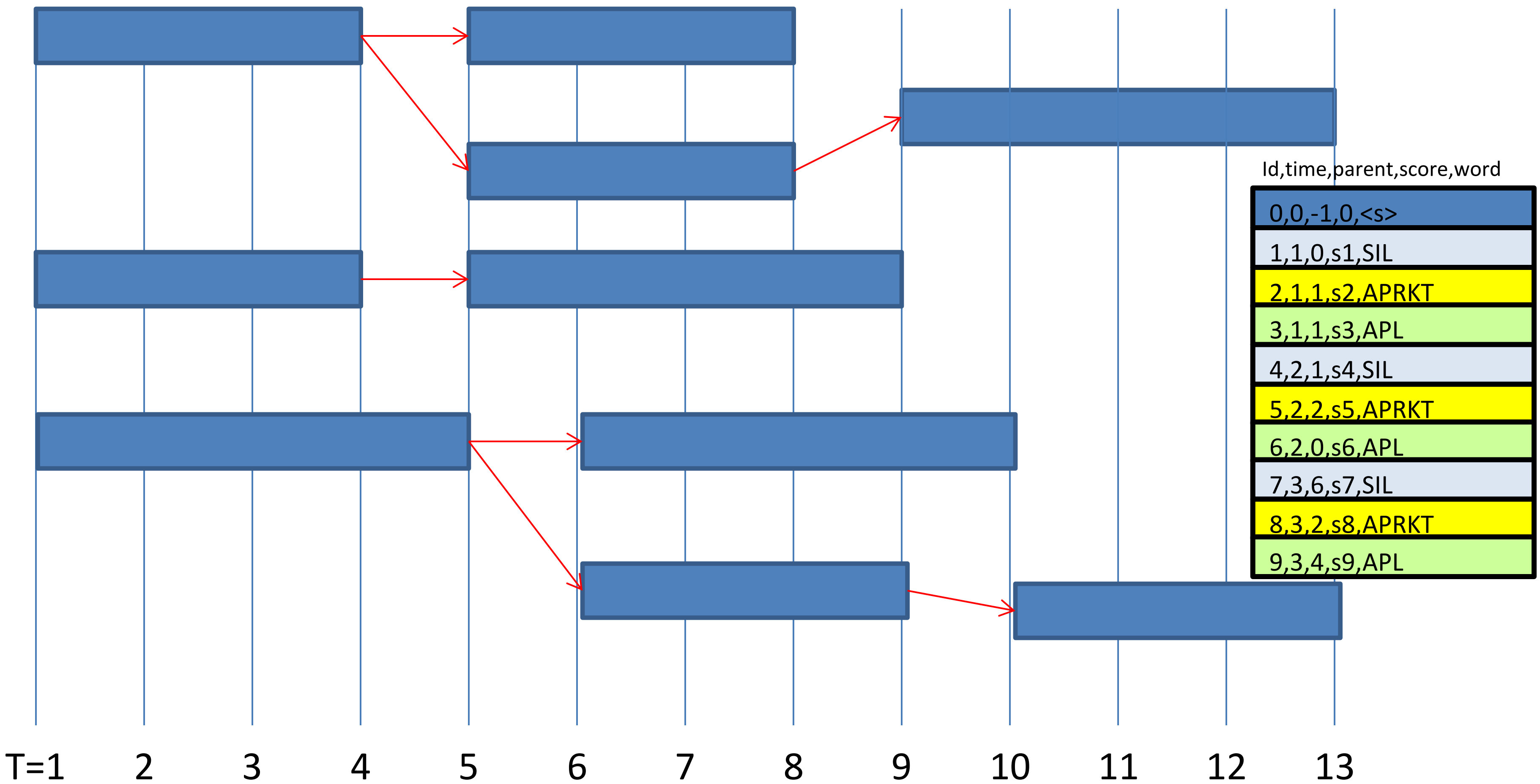
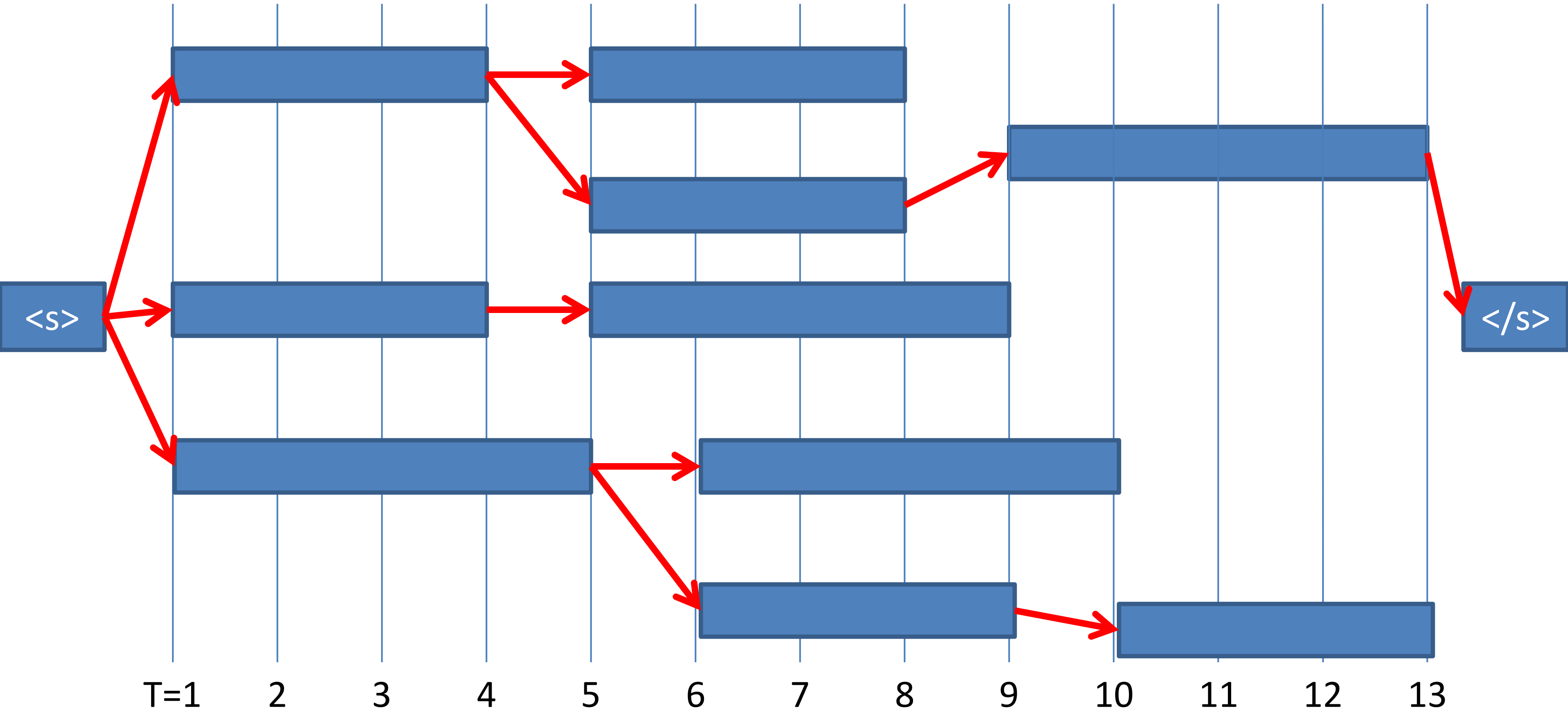
A different view of the BP table



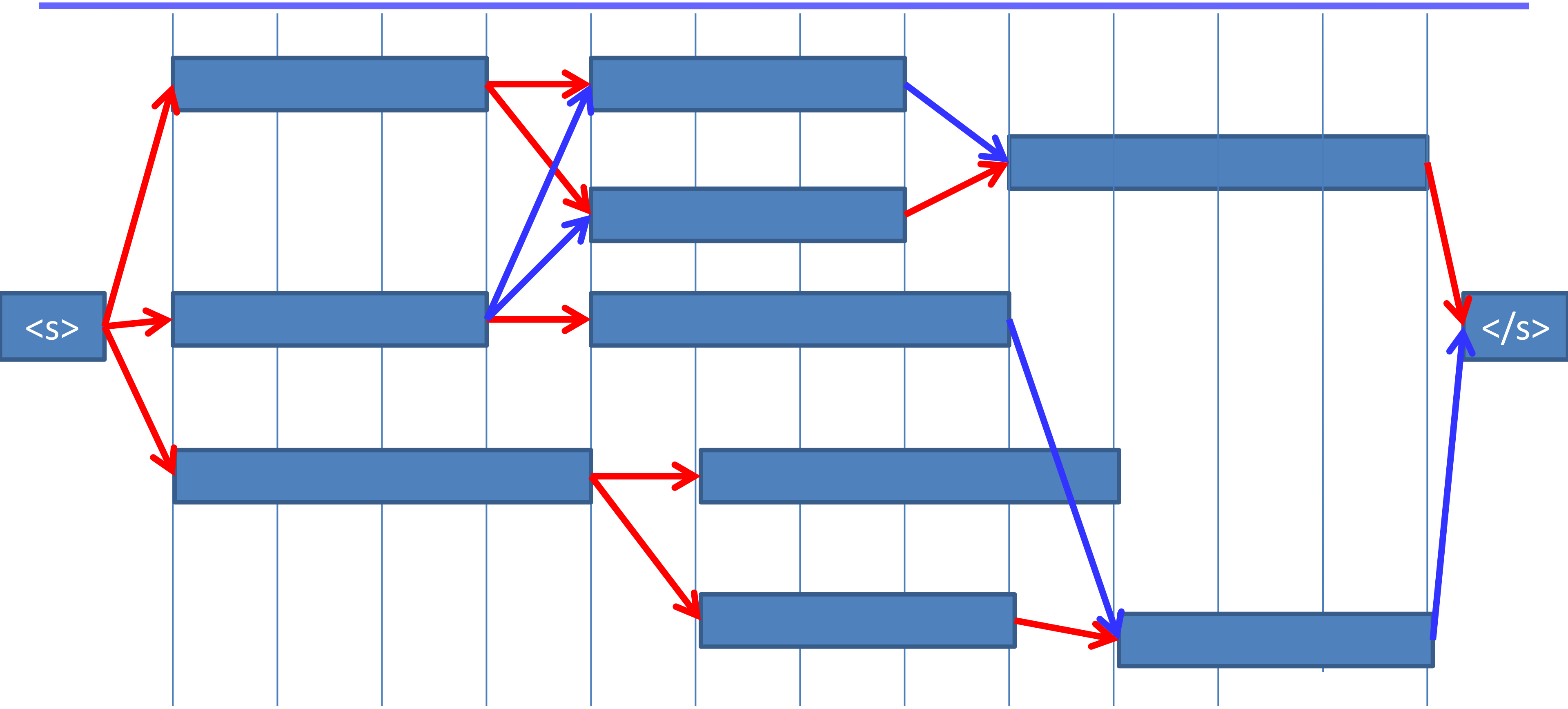
- Each rectangle is a BP table entry, with a start time, an end time, a word id, and a score. Some entries have no children

The BP Table as a Tree



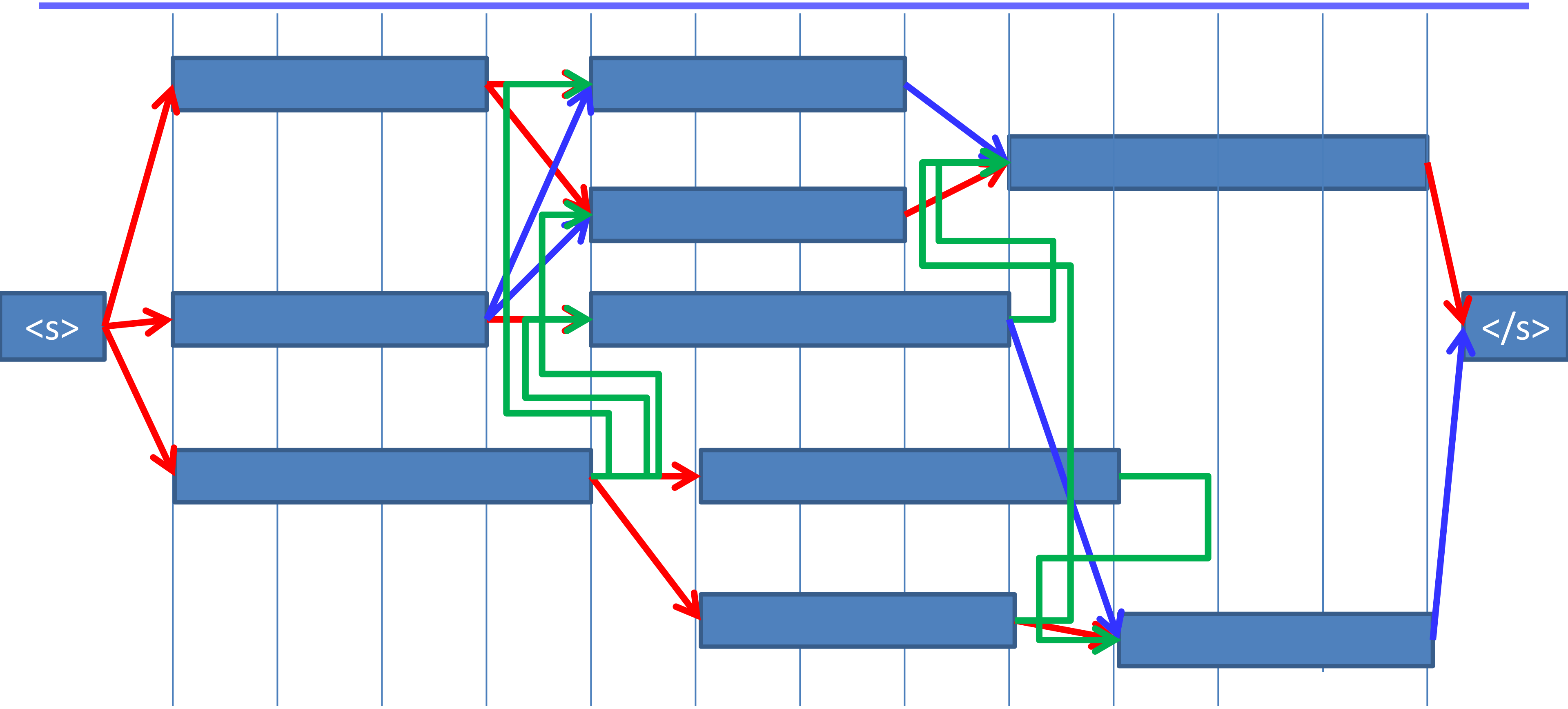
- **Introduce the begin-utterance and end-utterance markers**
- Note: Each node has a score
  - Acoustic score and LM score
  - Can be separated; Acoustic score stays at node, LM score rides incoming edge

The BP Table as a **DAG**



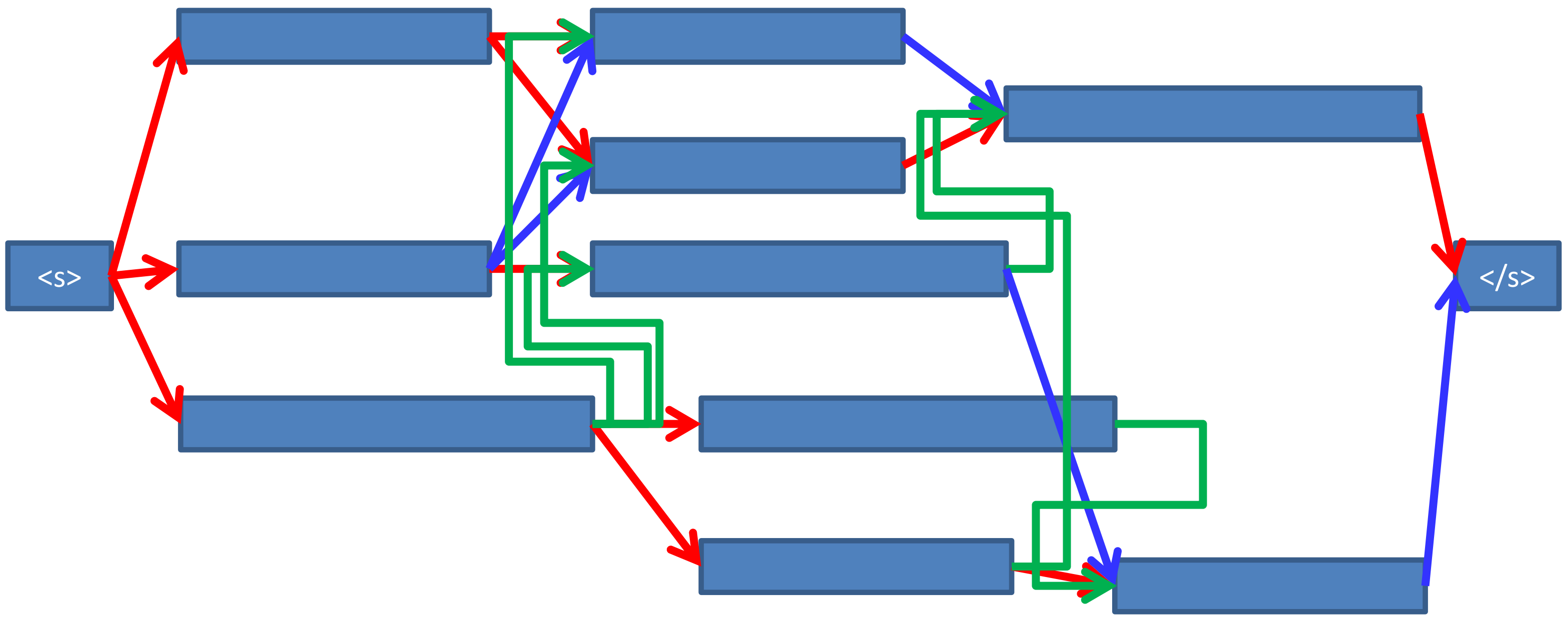
- **Add additional edges**
  - **Only between nodes whose timestamps match up**
  - **End frame of one nodes is immediately before first frame of next**
  - New edges can be assigned appropriate LM score
    - Or may be assigned a score via reasonable heuristics

The BP Table as a **DAG**



- “Approximate” edges
  - Add edges between nodes if they are only “slightly” misaligned
    - i.e. have a gap of less than X frames, or overlap by Y frames
      - Typical values: X = Y = 2.

The *LATTICE*



- The resulting structure is a DAG called the “LATTICE”
- It represents the set of all major word-sequence hypotheses that were “considered” by the recognizer
- It includes the final most-probable (best match) word sequence that was obtained, but also much more..
- Note: It’s a probabilistic structure
  - Each node and edge contain probability (or log prob) scores

Returning to ASR

- We now apply what we have learned to address some problems in speech recognition

- N-best generation
- Rescoring
- Confidence estimation

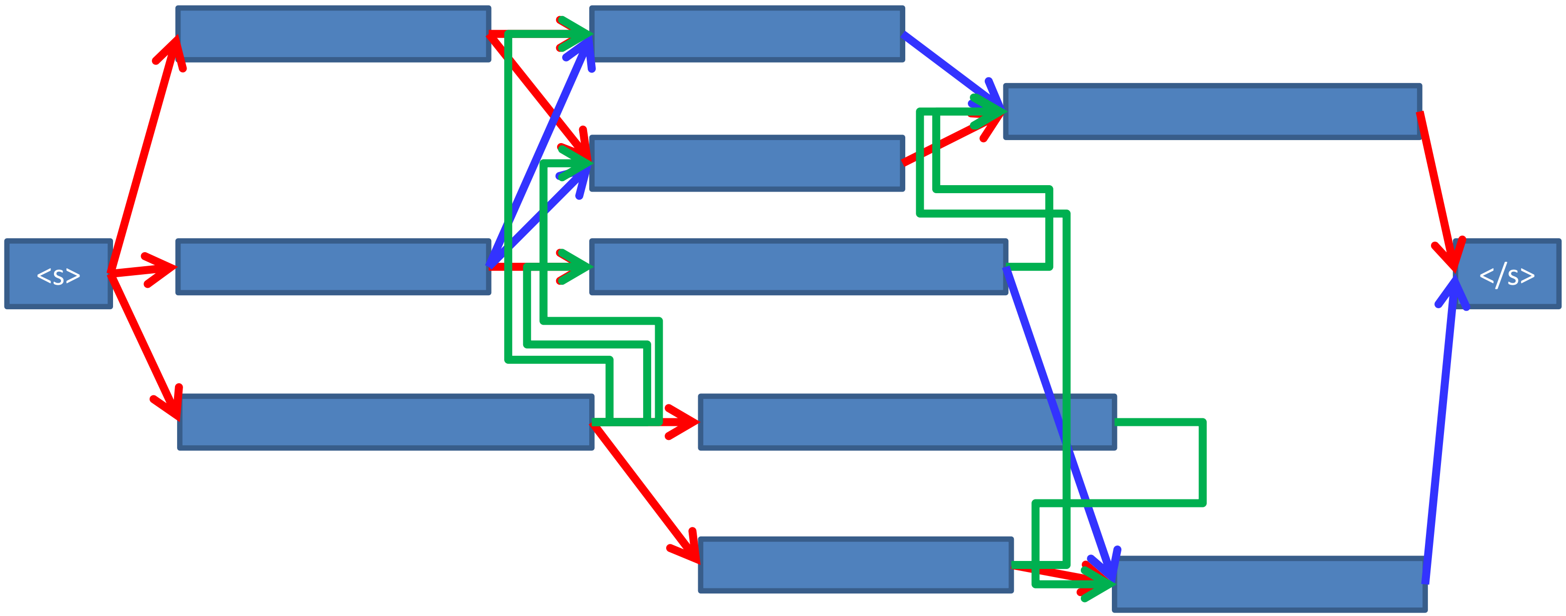
- We will perform all of this using the recognition lattice

Problem 1: N-best hypotheses

- The recognizer always outputs the *best-scoring* word sequence hypothesis
- What is the *second-best* scoring hypothesis?
- The *third-best* scoring hypothesis?
- The *Nth-best* scoring hypothesis?

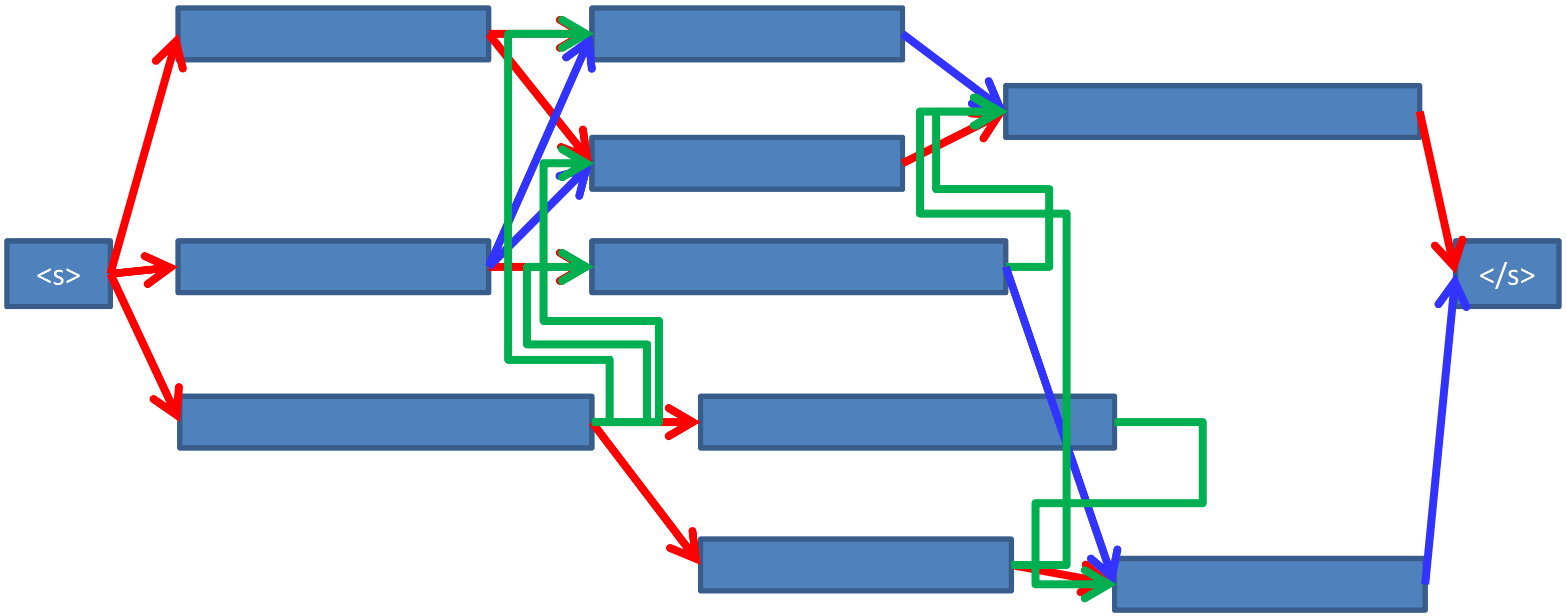
- The *N-best* output procedure

The Stack Decoder for N-best hypotheses



- Apply the stack decoding algorithm to obtain the N-best paths from <s> to </s>
  - Assuming single source node: <s>
  - Assuming one or more sink nodes: </s>

The A* Decoder for N-best hypotheses



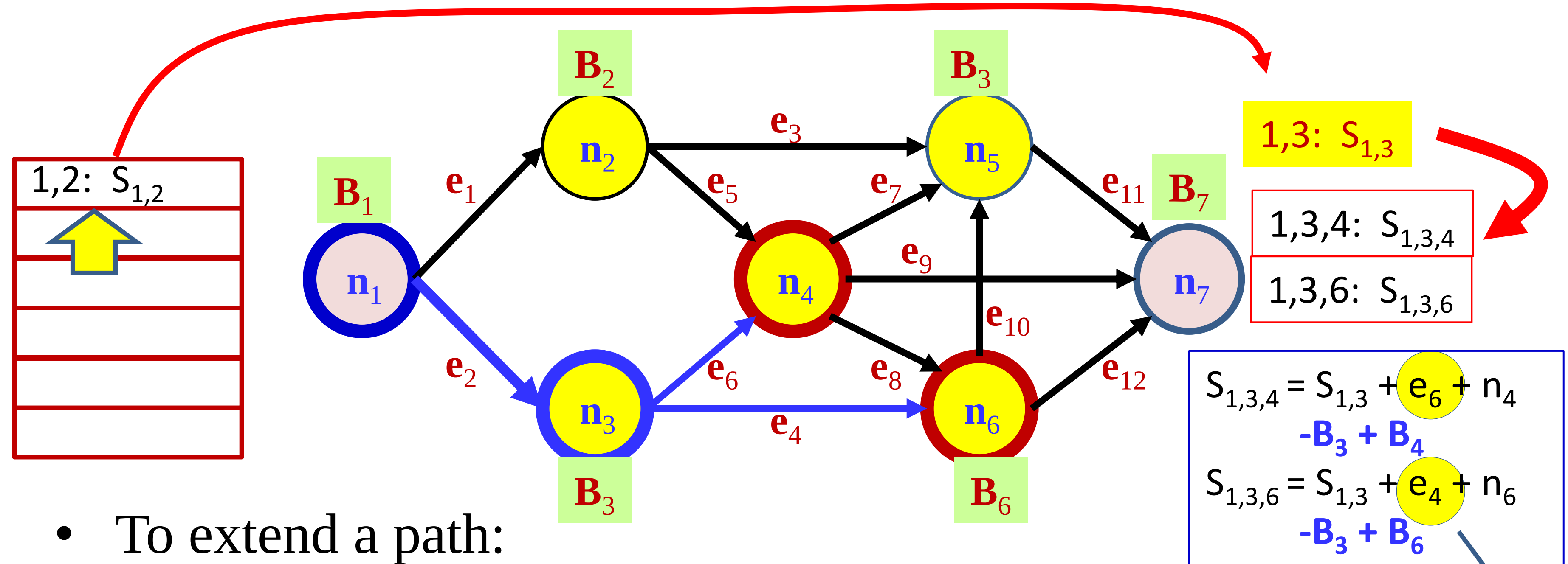
- Generally preferable to use the A* algorithm instead of the stack decoder
  - For reasons explained earlier

Rescoring: Using a different LM

- Common tactic:
  - Perform first pass of recognition with a “simple” language model
    - E.g. a bigram LM
    - Much more compact graph, much more efficient search
  - Find the best path through lattice using a higher-order (or more detailed) LM
  - Also called *Rescoring*

- Easily performed using a modification of the stack or A* decoder

*Rescoring through A**



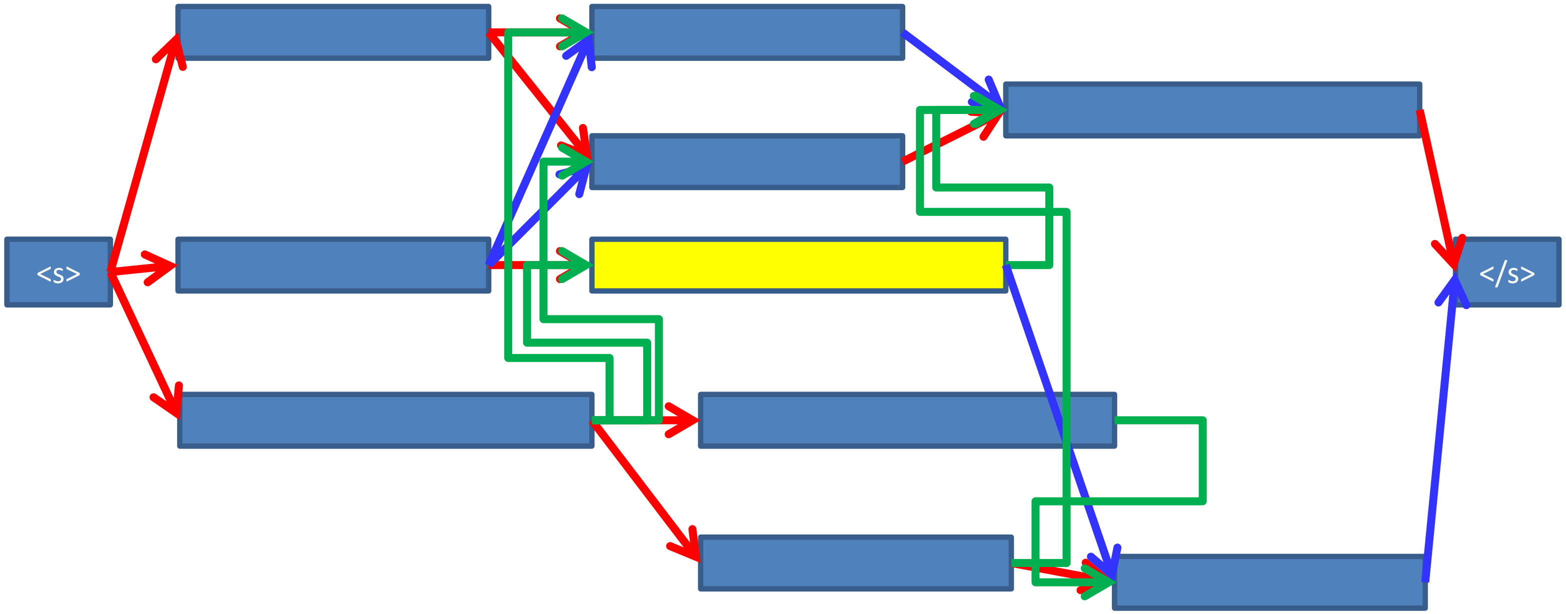
- To extend a path:
  - Append one-step extensions to current path
  - Factor in cost of one-step extensions
- The edge costs carry LM probabilities
- Refer to word history and obtain LM probabilities from new LM
  - Edge score = P(W(node) | Word history)

Confidence: How sure are we

- How sure can we be that the recognizer output is correct?
  - Often critical to know
  - If we are not confident, we must take corrective action

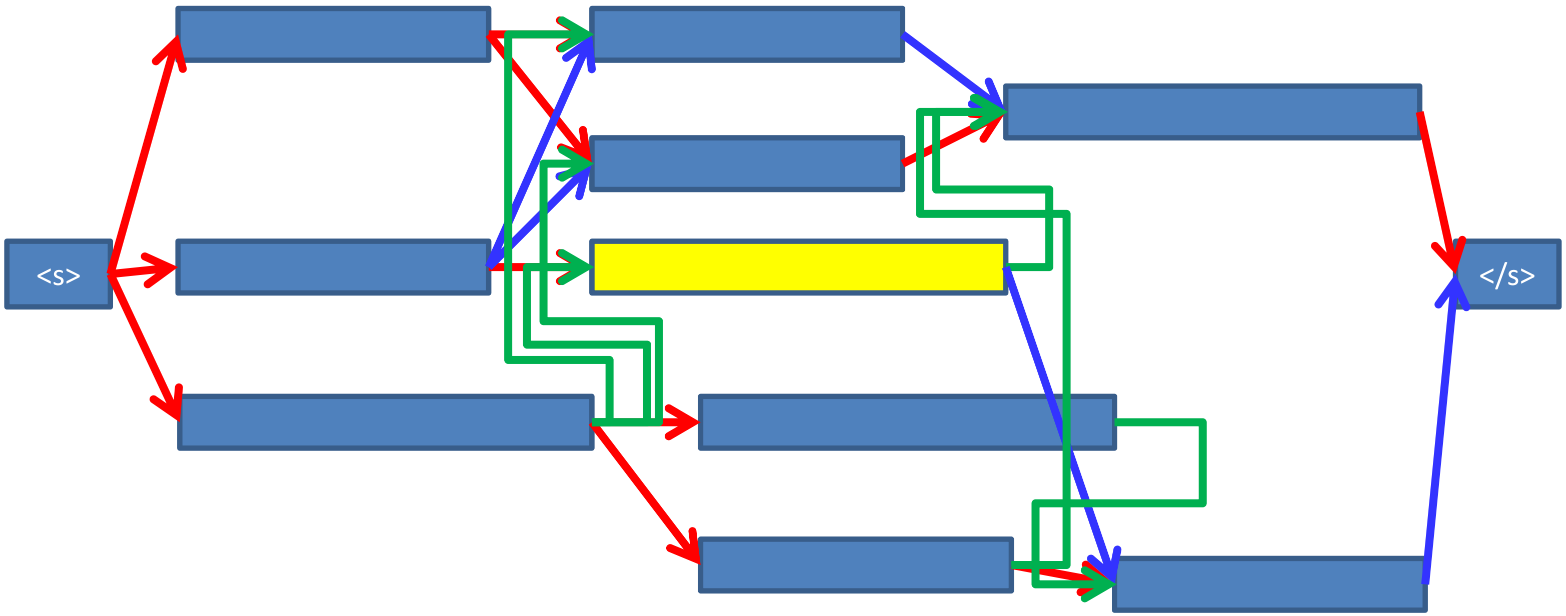
- No really robust method to compute confidence
- Confidence is often obtained as the *a posteriori* probability

Confidence as *a posteriori* probability



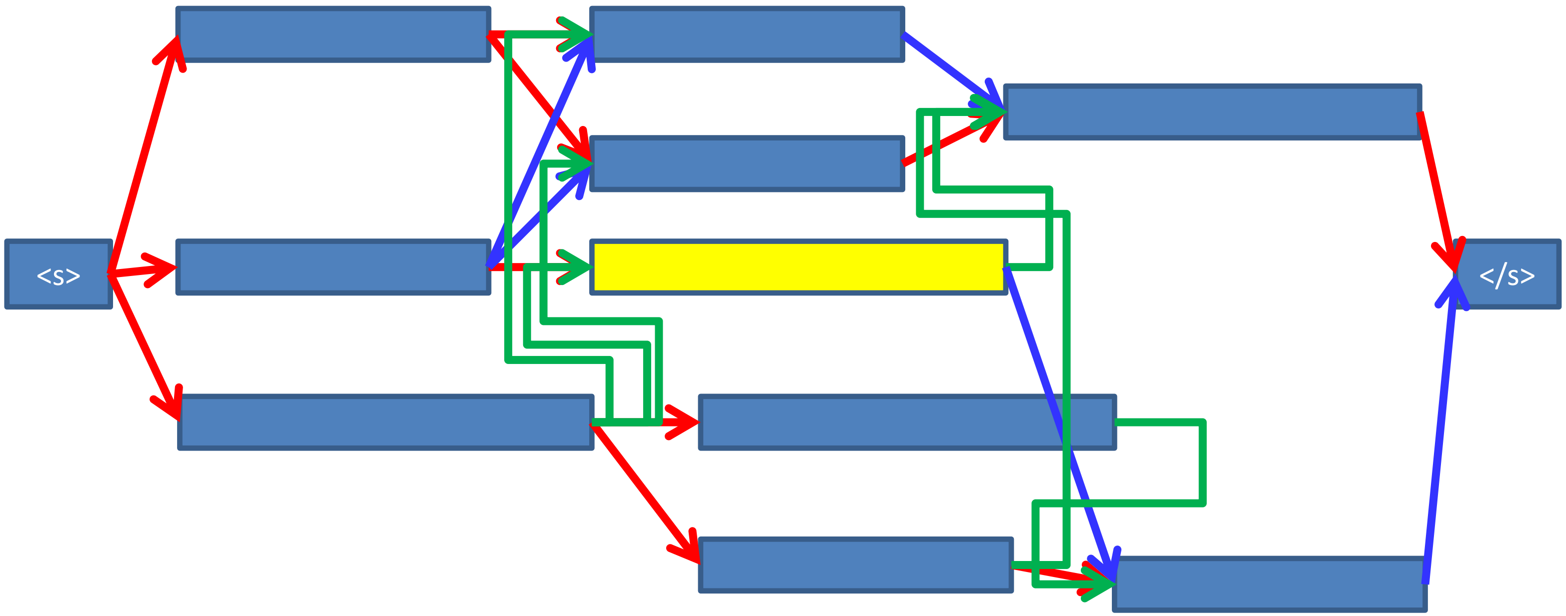
- Any word in our hypothesis represents a node in the lattice
- The confidence assigned to the word is the *a posteriori probability* of the node
  - A number between 0 and 1
  - 0 ➔ sure that its wrong; 1 ➔ sure that its correct
    - Caveat: We can be wrong when we're sure..

Computing *a posteriori* probability



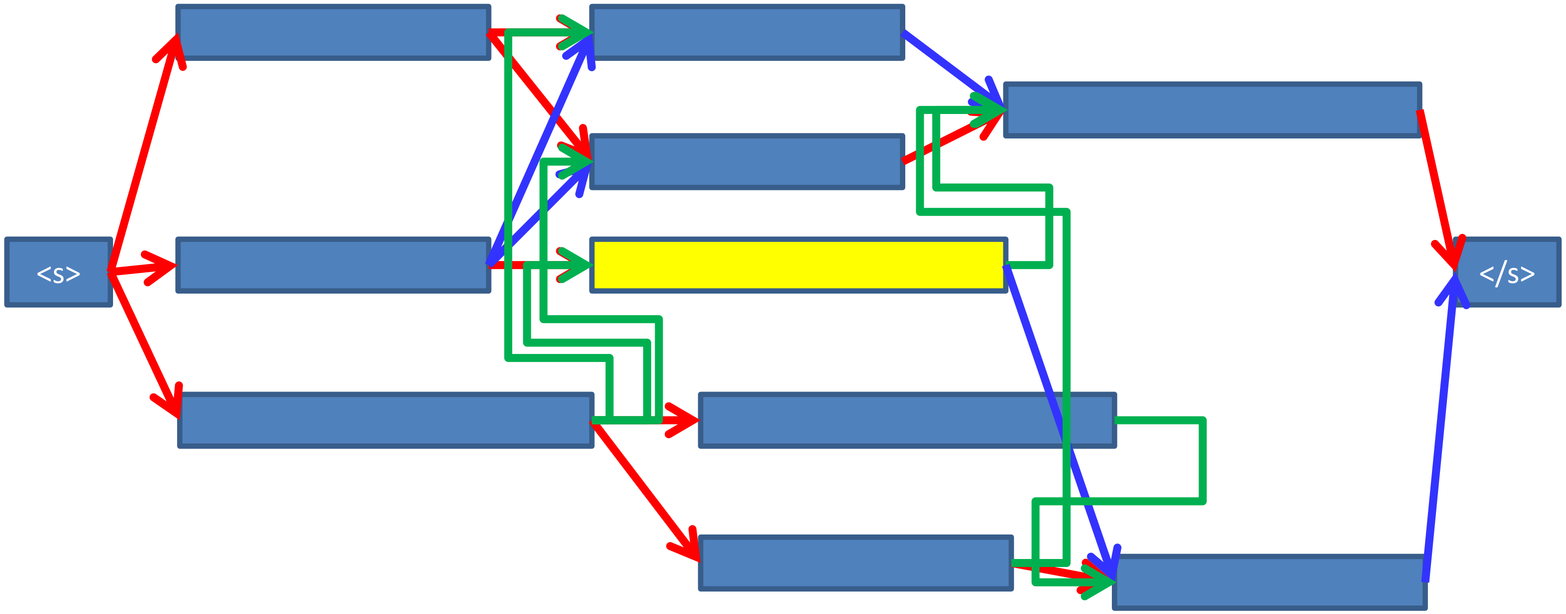
- A posteriori probability:
  - Total probability of all paths through node / total probability of graph
- We already know how to compute these

Assigning confidences



- For each node representing word in hypothesis: Compute total probability of all paths through node
  - Using forward-backward algorithm given earlier
- Compute total probability of graph
  - Also using the forward-backward algorithm

Assigning confidences



- For each node representing word in hypothesis:
  - Confidence = total prob of paths through node/ total prob
- Can in fact be computed for every node in the lattice

Topics covered

- N-best generation
- Rescoring
- Confidence estimation

- Using:
  - A*
    - Combines Dijkstra's algorithm and stack decoding
  - Forward-Backward algorithm

Additional topics

- Topics remaining:
- Improved confidence scoring
- *Acoustic* rescoring
- Adaptation
- Neural network methods