## Exact and Approximate Search for Automatic Speech Recognition

## Types of "Language Models"

- Finite state grammars
- The set of all possible word sequences is represented as a graph
- Context free grammars

- A set of context-free rules:
- Digit :=0|1|2;
- Number = Digit | Number Digit;
- Typically converted into a finite state graph for recognition
- Graph may be approximate

- Some CFGs are not representible as finite-state Graphs and require pushdown automata
- N -gram language models


## An Example Backoff Trigram LM

\1-grams:
-1.2041 <UNK> $\quad 0.0000$
$-1.2041</ s>\quad 0.0000$
$-1.2041<s>\quad-0.2730$
-0.4260 one -0.5283
-1.2041 three -0.2730
-0.4260 two -0.5283
\2-grams:
-0.1761 <s> one 0.0000

- 0.4771 one three 0.1761
$-\mathbf{0 . 3 0 1 0}$ one two $\mathbf{0 . 3 0 1 0}$
- 0.1761 three two 0.0000
- 0.3010 two one 0.3010
-0.4771 two three 0.1761
\3-grams:
-0.3010 <s> one two
- 0.3010 one three two
- 0.4771 one two one
- 0.4771 one two three
- 0.3010 three two one
-0.4771 two one three
-0.4771 two one two
-0.3010 two three two


## A COMPLETE TRIGRAM GRAPH



## A "Reduced" Trigram Graph

\1-grams:
-1.2041 <UNK> 0.0000
-1.2041 </s> 0.0000
-1.2041 <s> -0.2730
-0.4260 one -0.5283
-1.2041 three -0.2730
-0.4260 two
\2-grams:
$-0.1761<\mathrm{s}>$ one 0.0000
-0.4771 one three 0.1761

- 0.3010 one two 0.3010
-0.1761 three two 0.0000 - 0.3010 two one 0.3010 -0.4771 two three 0.1761 \3-grams:
$-0.3010<\mathrm{s}>$ on $<\mathrm{S}>(0)$
-0.3010 one three -0.4771 one two three -0.3010 three two one -0.4771 two one three -0.4771 two one two
-0.3010 two three two



## Ngrams: Can we do better

- Even reduced graphs can get very large
- Rarely directly used for recognition
- Alternate strategies must be employed
- Lextrees
- For low-order Ngrams only
- Approximate decoding strategies
- Lextrees + approximate decoding strategies
- Minimization strategies
- WFSTs: Using techniques from finite state automata theory


## A Unigram Graph



- Just a set of parallel word models with a loopback
- The ingoing edge into each word carries its LM probability

A Unigram Graph with words built from phonemes


- Composing Word models from phoneme models
- Each rectangle is actually an HMM. The entire graph is a large HMM


## A Unigram Lextree



- Eliminate redundancy in the graph
- But where do word probabilities get introduced?
- The identity of the word is not evident at entry!


## A Unigram Lextree with trailing probabilities



- Introduce word probabilities on the exit arcs
- The word identity is evident at that point


## A Unigram Lextree with spread probabilities



- Better still: Spread the probabilities
- Any arc that first identifies a subset of words carries the conditional probability of that subset


## A Bigram Graph



- Explicit connection from every word to every word
- Connections carry bigram probabilities


## A Bigram Graph:

## Adding silence



- Addition of looping silence is non-trivial
- What will the probability be on the outgoing edges from silence
- We do not have probabilities for P (word | silence), only P(word|word)
- If a silence occurs between two words, we use the word before the silence as context


## A Bigram Graph: Proper insertion of silences



- An explicit silence model at the end of every word
- We get an enormous number of copies of the silence model!


## What about Lextrees



- Can this be collapsed to a lextree?


## Probabilities on lextrees



- Word identities are not known on entry
- Only on word exit


## Probabilities on lextrees



- Word identities are not known on entry
- Only on word exit
- Word probabilities cannot be smeared
- Both word histories lead into the same node
- Uncertain which probability terms to use on inner connections


## Correct Lextrees



- Each edge carries the bigram probability of the exited word
- This is different from the "flat" structure where the edges carried probabilities of words to be entered
- All "Apple" exits enter lextree 1, all "apricot" exits enter lextree 2
- This graph is not complete: it ignores the first word in a


## Correct full lextrees

(</s>|aprocot)*P(apricot|apple)


- The word entry bigrams need their own lextree!
- Since neither of the second-level lextrees can represent a sentence-beginning context
- Lextree 1 represents the "Apple" context (only exits from the word "apple" enter this lextree
- Lextree 2 is the "apricot" context
- Why do transitions into the end of sentence have products of two probability terms?


## Correct full lextrees with silence



- Fortunately, adding silence doesn't complicate this too much
- Add a looping silence at the beginning of each lextree
- And one at the sentence terminator


## Correct Structures are Limiting

- The "correct" flat N-gram structure can get very large
- $\mathrm{D}+\mathrm{D}^{2}+. .+\mathrm{D}^{\mathrm{N}-1}$ word HMMs are required in the larger "Language" HMM
- Even the reduced N -gram structure can be very large
- Reduced structures are not exact
- Multiple paths exist for each N-gram
- Reduced structures are nevertheless used very effectively by WFSTbased strategies
- Lextrees result in significant compression for Unigram LMs
- But for N-gram LMs "correct" Lextree-based graphs are much larger than "flat" graphs
- Need $\mathrm{D}+\mathrm{D}^{2}+. .+\mathrm{D}^{\mathrm{N}-1}$ lextrees!!


## Approximate Search Strategies

- Approximate search strategies are not guaranteed to result in the best recognition
- Although, in practice they often approach the optimal recognition
- Efficiency is obtained by dynamically modifying graph parameters
- I.e. language probabilities in the language HMM
- This is typically done by utilizing word histories
- From a backpointer table
- The resulting improvement in efficiency can be very very large


## Approximate search with a Unigram Lextree



- Utilize the above lextree as the basic HMM structure
- Note - no language model probabilities are loaded on the lextree
- These will be imposed dynamically during search
- In practice unigram probabilities may be factored into the lextree and factored out during search
- We will ignore this option in the following explanation


## Approximate search with a Unigram Lextree



- We will use the simplified figure above in the following explanation
- AEP is the concatenation of AE and P
- AXL is the concatenation of $A X$ and $L$
- RAKT is the concatenation of R AX K AA and T
- Will not explicitly show silence models


## Approximate search with a Unigram Lextree



- A Linear Representation that is useful to draw a trellis
- Note: Each box is actually an HMM (representing a sequence of phonemes)
- For simplicity we will assume each box has only one state


## Approximate search Trellis



- A normal unigram trellis, but with no LM probabilities at word transitions


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## Approximate search Trellis



- Search follows usual rules except that at word transitions we look up the word history to apply LM probabilities


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We will actually use $\log ($ LMPROB ) as edge score during search


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## Approximate search Trellis

| $0,0,-1,0,<s>$ |
| :---: |
| $1,1, \mathrm{~s} 1,0$, apple |



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## Approximate search Trellis

| $0,0,-1,0,<s>$ |
| :---: |
| 1,1, s1,0,apple |
| 2,2, s2,0,apricot |



- Search follows usual rules except that at word transitions we look up the word history to apply LM probabilities

Approximate search Trellis

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| 2,2, s2,0, apricot |



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## Approximate search Trellis

| $0,0,-1,0,<s>$ |
| :---: |
| 1,1, s 1,0, apple |
| 2,2, s2,0, apricot |



* The transition out of "Apricot" carries the probability P(Apricot|Apple) because the parent of the current state is the word "apple"
- This information is retrieved from the backpointer table

Approximate search Trellis

| $0,0,-1,0$, <s> |
| :---: |
| 1,1, s 1,0, apple |
| 2,2, s 2,0, apricot |



- Search rules do not change - the best incoming entry is retained

Approximate search Trellis


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Approximate search Trellis


- Note the conditioning word in the bigram probabilities applied


## Approximate search Trellis

| $0,0,-1,0,<s>$ |
| :---: |
| $1,1, \mathrm{~s} 1,0$, apple |
| 2,2, s2,0,apricot |
| $3,3, \mathrm{~s} 3,1$, apricot |

- The winner remains as before


## Approximate search Trellis



- The winner remains as before


## Approximate search Trellis



- Lets follow this to the end


## Approximate search Trellis



- Lets follow this to the end

Approximate search Trellis


- Lets follow this to the end

- Note the probabilities being applied to the final transition into sentence ending!


## Approximate structures with lextrees

- Can use trigram probabilities instead of bigrams without modifying search strategy
- Determine previous TWO words and apply appropriate LM trigram probability during search
- Can in fact handle ANY left-to-right language model
- The approximate structure shown earlier is suboptimal
- Although highly popular, particularly for embedded systems
- A better approximation is obtained using multiple lextrees
- Typically 3-5 lextrees
- The distinction between the lextrees is in the time of entry: incoming arcs into the j -th ( of K ) lextrees only activate if $\mathrm{T} \% \mathrm{~K}=\mathrm{j}$
- i.e. each lextree can be entered only once every K frames
- Other similar heuristics may be applied
- A still better approximation is obtained using a flat bigram search structure


## Approximate decode with flat bigram structure



- A better (but more complex) approximate search uses the flat bigram structure shown above
- Note the manner in which silence is inserted
- Very simple
- Once again, no LM probabilities are introduced at this stage


## A closer look at the flat bigram



- Not showing silence above to keep it simple
- But in reality, silence will be included
- Note: No LM probabilities included
- We take no advantage of the fact that phonemes are shared, however
- We want to be able to determine word identity at the entry to a word
- In the following slides we will not show the phonetic breakup of words to keep figures simple


## The flat bigram structure



- In the following slides we will assume each word has only one state to simplify illustration


## Recognition with flat bigram structure



- The trellis is composed as usual
- But no cross-word language-probabilities are introduced
- Note: In this form of trellis the non-emitting state at word beginning may be superfluous


## Recognition with flat bigram structure



- Bigram probabilities conditioned on start of sentence are applied at the beginning
- Entries to silence carry silence penalty


## Recognition with flat bigram structure



- Word ending states move into the backpointer table


## Recognition with flat bigram structure



- Word ending states move into the backpointer table


## Recognition with flat bigram structure

Some arcs have bigram probs, others have trigram probs, and yet others have none


For search we actually use $\log (L M P R O B)$ as edge score


- Note the different LM probability terms applied to the arcs
- Assuming trigram LM
- The appropriate history to use for the LM probability is obtained from the BPtable


## Recognition with flat bigram structure

Some arcs have bigram probs, others have trigram probs, and yet others have none


For search we actually use $\log ($ LMPROB) as edge score


- Note the different LM probability terms applied to the arcs
- Assuming trigram LM
- The appropriate history to use for the LM probability is obtained from the BPtable


## Recognition with flat bigram structure



- All cross-word arcs into SILENCE carry the silence penalty
- Self-transitions within the silence will only carry the self-transition probability for the states of the Silence model


## Recognition with flat bigram structure



- The actual computation evaluates all of these states in the same timestep


## Recognition with flat bigram structure



- The actual computation evaluates all of these states in the same timestep


## Recognition with flat bigram structure



- The actual computation evaluates all of these states in the same timestep


## Recognition with flat bigram structure



- Word ending states move into the BP table


## Recognition with flat bigram structure



- Word ending states move into the BP table


## Cross-word Pruning

- We can apply a second pruning threshold locally to all entries added to the BP table at a given time
- This is the "new-word beam"
- This is different from the state-level beam applied across all active states at a given time
- This is only applied to new word terminations
- A similar new-word beam may also be applied to the approximate lextree and to correct flat and lex-tree graphs
- In other words, there are TWO different beams we will apply
- A state-level beam to prune poorly-scoring states
- A word-level beam to prune poorly-scoring words
- Word beams are typically narrower than state beams


## Recognition with flat bigram structure



- Pruning the word exits


## Recognition with flat bigram structure



- Note the different LM probabilities applied


## Recognition with flat bigram structure



- Select the "winner"


## Recognition with flat bigram structure



- Note the different LM probabilities applied


## Recognition with flat bigram structure



- As before, word ending states move into the BP table


## Recognition with flat bigram structure



- As before, word ending states move into the BP table


## Recognition with flat bigram structure



- As before, word ending states move into the BP table
- And pruned


## Recognition with flat bigram structure



- Note LM probabilities now


## Recognition with flat bigram structure



## Recognition with flat bigram structure



- Note LM probabilities now


## Recognition with flat bigram structure



## Recognition with flat bigram structure



- Note LM probabilities now


## Recognition with flat bigram structure



- Note LM probabilities now


## Recognition with flat bigram structure



- These word exits will end up in the BP table (not shown)


## Recognition with flat bigram structure



- These word exits will end up in the BP table (not shown)


## Recognition with flat bigram structure



- Note Sentence Ending LM Probabilities Used
- Note also that multiple hypotheses represent the same word sequence
- Varying only in the location of silences and word boundaries


## Additional Issues

- Several topics left uncovered
- We lost 3 weeks
- Multi-pass search strategy:
- The BP table is actually a "lattice"
- A graph of words
- A common strategy is to compute a lattice using a bigram LM and to use that as a grammar/graph for recognition using higher-order N -gram LMs
- N-best hypotheses generation
- How to search the word graph to generate more than one hypotheses
- Confidence: How to assign a "confidence" score to a hypothesis
- How much we believe the recognizer's output


## Final Assignment

- N-gram based recognition using an approximate decoding strategy
- Choose between lextree and flat bigram structure
- We are still the equivalent of 4-5 assignments from a nearly-state-of-art system
- Triphones
- Lattices
- Rescoring
- Nbest
- Confidence
- In reality, each step would have been very incremental..

